

Задача № 1

Заданы векторы  $\bar{a}, \bar{b}, \bar{c}$  своими декартовыми координатами.

Требуется:

3

- 1) найти линейную комбинацию  $\bar{a} + 2\bar{b} - \bar{c}$  и построить результирующий вектор;
- 2) найти  $np_{\bar{i}}(\bar{a} + 2\bar{b}), np_{\bar{j}}(\bar{a} + 2\bar{b}), np_{\bar{k}}(\bar{a} + 2\bar{b})$ ;
- 3) проверить, коллинеарны ли векторы  $\bar{a} + 2\bar{b}$  и  $\bar{a} + \bar{c}$ ;
- 4) проверить, ортогональны ли векторы  $\bar{a}$  и  $\bar{c}$ ;
- 5) найти скалярное произведение  $(2\bar{a} - \bar{b})\bar{c}$ ;
- 6) найти векторное произведение  $(3\bar{a} - 2\bar{b}) \times (\bar{a} + \bar{c})$ ;
- 7) найти  $орт \bar{n}^0$ , ортогональный векторам  $\bar{a}$  и  $\bar{b}$ ;
- 8) проверить, компланарны ли векторы  $\bar{a}, \bar{b}, \bar{c}$ .

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|-------------------------------|----------------------------|---------------------------|
| 1. $\bar{a} = \{1; -2; 3\},$  | $\bar{b} = \{3; 0; -1\},$  | $\bar{c} = \{3; 1; -2\}$  |
| 2. $\bar{a} = \{1; 0; 1\},$   | $\bar{b} = \{-2; 3; 5\},$  | $\bar{c} = \{0; 2; 1\}$   |
| 3. $\bar{a} = \{-2; 4; 1\},$  | $\bar{b} = \{1; -2; 7\},$  | $\bar{c} = \{5; -3; 2\}$  |
| 4. $\bar{a} = \{1; 2; -3\},$  | $\bar{b} = \{2; -1; -1\},$ | $\bar{c} = \{3; -1; 2\}$  |
| 5. $\bar{a} = \{3; 5; 4\},$   | $\bar{b} = \{5; 9; 7\},$   | $\bar{c} = \{1; -1; 2\}$  |
| 6. $\bar{a} = \{1; 4; -2\},$  | $\bar{b} = \{1; 1; -1\},$  | $\bar{c} = \{4; 1; -2\}$  |
| 7. $\bar{a} = \{1; -2; 5\},$  | $\bar{b} = \{3; -1; 0\},$  | $\bar{c} = \{2; -1; 4\}$  |
| 8. $\bar{a} = \{3; 4; -1\},$  | $\bar{b} = \{2; -1; 1\},$  | $\bar{c} = \{1; 1; -2\}$  |
| 9. $\bar{a} = \{2; -2; 1\},$  | $\bar{b} = \{1; 0; 5\},$   | $\bar{c} = \{-1; 2; 1\}$  |
| 10. $\bar{a} = \{-1; 4; 2\},$ | $\bar{b} = \{3; -2; 6\},$  | $\bar{c} = \{2; 0; -1\}$  |
| 11. $\bar{a} = \{0; 3; -2\},$ | $\bar{b} = \{1; -2; 1\},$  | $\bar{c} = \{2; 1; -1\}$  |
| 12. $\bar{a} = \{-2; 7; 1\},$ | $\bar{b} = \{-3; 5; 2\},$  | $\bar{c} = \{4; 1; 1\}$   |
| 13. $\bar{a} = \{3; 7; 0\},$  | $\bar{b} = \{1; -3; 4\},$  | $\bar{c} = \{1; 3; -1\}$  |
| 14. $\bar{a} = \{-1; 2; 1\},$ | $\bar{b} = \{2; -7; 1\},$  | $\bar{c} = \{2; -1; 3\}$  |
| 15. $\bar{a} = \{7; 1; -2\},$ | $\bar{b} = \{5; 4; 3\},$   | $\bar{c} = \{0; -1; 2\}$  |
| 16. $\bar{a} = \{5; 0; -2\},$ | $\bar{b} = \{3; 4; -1\},$  | $\bar{c} = \{3; 1; -1\}$  |
| 17. $\bar{a} = \{2; 3; -1\},$ | $\bar{b} = \{4; 1; 3\},$   | $\bar{c} = \{1; 0; -1\}$  |
| 18. $\bar{a} = \{3; -1; 2\},$ | $\bar{b} = \{5; 7; 10\},$  | $\bar{c} = \{-1; 0; 2\}$  |
| 19. $\bar{a} = \{1; -2; 4\},$ | $\bar{b} = \{7; 3; 5\},$   | $\bar{c} = \{0; 1; -1\}$  |
| 20. $\bar{a} = \{3; 7; 0\},$  | $\bar{b} = \{4; 2; -1\},$  | $\bar{c} = \{2; -1; -1\}$ |

### Задача № 2

Заданы вершины тетраэдра  $A(X_A; Y_A; Z_A)$ ,  $B(X_B; Y_B; Z_B)$ ,  $C(X_C; Y_C; Z_C)$ ,  $D(X_D; Y_D; Z_D)$ .

Требуется найти:

- 1) угол между ребрами  $AB$  и  $AD$ ;
- 2)  $np_{AC} \overline{AD}$ ;
- 3) площадь грани  $ABC$ ;
- 4) объем тетраэдра;
- 5) высоту тетраэдра, опущенную из вершины  $D$  на грань  $ABC$ .

|                     |                  |                 |                 |
|---------------------|------------------|-----------------|-----------------|
| 1. $A(1; 3; 4)$ ,   | $B(2; 2; 1)$ ,   | $C(-1; 0; 1)$   | $D(-4; 6; -3)$  |
| 2. $A(-4; 2; 6)$ ,  | $B(2; -3; 0)$ ,  | $C(-10; 5; 8)$  | $D(-5; 2; -4)$  |
| 3. $A(7; 2; 4)$ ,   | $B(7; -1; -2)$ , | $C(3; 3; 1)$    | $D(-4; 2; 1)$   |
| 4. $A(2; 1; 4)$ ,   | $B(-1; 5; -2)$ , | $C(-7; -3; 2)$  | $D(-6; -3; 6)$  |
| 5. $A(-1; -5; 2)$ , | $B(-6; 0; -3)$ , | $C(3; 6; -3)$   | $D(-10; 6; 7)$  |
| 6. $A(0; -1; -4)$ , | $B(-2; 3; 5)$ ,  | $C(1; -5; -9)$  | $D(-1; -6; 3)$  |
| 7. $A(5; 2; 0)$ ,   | $B(2; 5; 0)$ ,   | $C(1; 2; 4)$    | $D(-1; 1; 1)$   |
| 8. $A(2; -1; -2)$ , | $B(1; 2; 1)$ ,   | $C(5; 0; -6)$   | $D(-10; 9; -7)$ |
| 9. $A(-2; 0; -4)$ , | $B(-1; 7; 1)$ ,  | $C(4; -8; 4)$   | $D(1; -4; 6)$   |
| 10. $A(14; 4; 5)$ , | $B(-5; -3; 2)$ , | $C(-2; -6; -3)$ | $D(-2; 2; -1)$  |
| 11. $A(1; 2; 1)$ ,  | $B(3; 0; -3)$ ,  | $C(5; 2; 6)$    | $D(8; 4; -9)$   |
| 12. $A(2; -1; 2)$ , | $B(1; 2; -1)$ ,  | $C(3; 2; 1)$    | $D(-4; 2; 5)$   |
| 13. $A(1; 1; 2)$ ,  | $B(-1; 1; 3)$ ,  | $C(2; -2; 4)$   | $D(-1; 0; -2)$  |
| 14. $A(2; 3; 1)$ ,  | $B(4; 1; -2)$ ,  | $C(6; 3; 7)$    | $D(7; 5; -3)$   |
| 15. $A(1; 1; -1)$ , | $B(2; 3; 1)$ ,   | $C(3; 2; 1)$    | $D(5; 9; -8)$   |
| 16. $A(1; 5; -7)$ , | $B(-3; 6; 3)$ ,  | $C(-2; 7; 3)$   | $D(-4; 8; -12)$ |

|                       |                   |                |                 |
|-----------------------|-------------------|----------------|-----------------|
| 17. $A(-3; 4; -7)$ ,  | $B(1; 5; -4)$ ,   | $C(-5; -2; 0)$ | $D(2; 5; 4)$    |
| 18. $A(-1; 2; -3)$ ,  | $B(4; -1; 0)$ ,   | $C(2; 1; -2)$  | $D(3; 4; 5)$    |
| 19. $A(4; -1; 3)$ ,   | $B(-2; 1; 1)$ ,   | $C(0; -5; 1)$  | $D(3; 2; -6)$   |
| 20. $A(1; -1; 1)$ ,   | $B(-2; 0; 3)$ ,   | $C(2; 1; -1)$  | $D(2; -2; -4)$  |
| 21. $A(1; 2; 0)$ ,    | $B(1; -1; 2)$ ,   | $C(0; 1; -1)$  | $D(-3; 0; 1)$   |
| 22. $A(1; 0; 2)$ ,    | $B(1; 2; -1)$ ,   | $C(2; -2; 1)$  | $D(2; 1; 0)$    |
| 23. $A(1; 2; -3)$ ,   | $B(1; 0; 1)$ ,    | $C(-2; -1; 6)$ | $D(0; -5; -4)$  |
| 24. $A(3; 10; -1)$ ,  | $B(-2; 3; -5)$ ,  | $C(-6; 0; -3)$ | $D(1; -1; 2)$   |
| 25. $A(-1; 2; 4)$ ,   | $B(-1; -2; -4)$ , | $C(3; 0; -7)$  | $D(7; -3; 1)$   |
| 26. $A(0; -3; 1)$ ,   | $B(-4; 1; 2)$ ,   | $C(2; -1; 5)$  | $D(3; 1; -4)$   |
| 27. $A(1; 3; 0)$ ,    | $B(4; -1; 2)$ ,   | $C(3; 0; 1)$   | $D(-4; 3; 5)$   |
| 28. $A(-2; -1; -1)$ , | $B(0; 3; 2)$ ,    | $C(3; 1; -4)$  | $D(-4; 7; 3)$   |
| 29. $A(-3; -5; 6)$ ,  | $B(2; 1; -4)$ ,   | $C(0; -3; -1)$ | $D(-5; 2; -8)$  |
| 30. $A(2; -4; -3)$ ,  | $B(5; -6; 0)$ ,   | $C(-1; 3; -3)$ | $D(-10; -8; 7)$ |

**Задача № 3**

1. Проверить, что вектора  $\bar{p}, \bar{q}, \bar{r}$  образуют базис в пространстве.
2. Найти разложение вектора  $\bar{a}$  по этому базису.

1.  $\bar{a} = \{-2; 5; 9\}, \quad \bar{p} = \{0; 1; 2\}, \quad \bar{q} = \{1; 0; 1\}, \quad \bar{r} = \{-1; 2; 4\}$
2.  $\bar{a} = \{-6; 11; 3\}, \quad \bar{p} = \{0; 5; 1\}, \quad \bar{q} = \{3; 2; -1\}, \quad \bar{r} = \{-1; 1; 0\}$
3.  $\bar{a} = \{1; -8; 4\}, \quad \bar{p} = \{1; 0; 1\}, \quad \bar{q} = \{0; -2; 1\}, \quad \bar{r} = \{1; 3; 0\}$
4.  $\bar{a} = \{1; 0; 6\}, \quad \bar{p} = \{2; 0; 1\}, \quad \bar{q} = \{1; -1; 0\}, \quad \bar{r} = \{-3; 2; 5\}$
5.  $\bar{a} = \{3; 0; 4\}, \quad \bar{p} = \{2; 1; 0\}, \quad \bar{q} = \{1; 0; 1\}, \quad \bar{r} = \{4; 2; 1\}$
6.  $\bar{a} = \{0; 8; 7\}, \quad \bar{p} = \{0; 3; 1\}, \quad \bar{q} = \{1; -1; 2\}, \quad \bar{r} = \{2; -1; 0\}$
7.  $\bar{a} = \{-1; -4; 10\}, \quad \bar{p} = \{1; -1; 2\}, \quad \bar{q} = \{3; 2; 0\}, \quad \bar{r} = \{-1; 1; 1\}$
8.  $\bar{a} = \{9; 10; 2\}, \quad \bar{p} = \{1; 1; 4\}, \quad \bar{q} = \{-3; 0; 2\}, \quad \bar{r} = \{1; 2; -1\}$
9.  $\bar{a} = \{-1; -7; 2\}, \quad \bar{p} = \{0; -2; 1\}, \quad \bar{q} = \{3; 1; -1\}, \quad \bar{r} = \{4; 0; 1\}$
10.  $\bar{a} = \{7; 1; 8\}, \quad \bar{p} = \{0; 1; 5\}, \quad \bar{q} = \{3; -1; 2\}, \quad \bar{r} = \{-1; 0; 1\}$
11.  $\bar{a} = \{8; -3; 6\}, \quad \bar{p} = \{1; 0; 1\}, \quad \bar{q} = \{1; -2; 0\}, \quad \bar{r} = \{0; 3; 1\}$
12.  $\bar{a} = \{-4; 4; 7\}, \quad \bar{p} = \{1; 3; 0\}, \quad \bar{q} = \{2; -1; 1\}, \quad \bar{r} = \{0; -1; 2\}$
13.  $\bar{a} = \{3; -3; 8\}, \quad \bar{p} = \{2; 1; -1\}, \quad \bar{q} = \{0; 3; 2\}, \quad \bar{r} = \{1; -1; 1\}$

14.  $\bar{a} = \{6; -1; 13\}, \quad \bar{p} = \{4; 1; 1\}, \quad \bar{q} = \{2; 0; -3\}, \quad \bar{r} = \{-1; 2; 1\}$
15.  $\bar{a} = \{-8; 10; 7\}, \quad \bar{p} = \{-2; 0; 1\}, \quad \bar{q} = \{1; 3; -1\}, \quad \bar{r} = \{0; 4; 4\}$
16.  $\bar{a} = \{3; 3; 2\}, \quad \bar{p} = \{5; 1; 0\}, \quad \bar{q} = \{2; -1; 3\}, \quad \bar{r} = \{1; 0; -1\}$
17.  $\bar{a} = \{8; 7; 4\}, \quad \bar{p} = \{0; 1; 1\}, \quad \bar{q} = \{-2; 0; 1\}, \quad \bar{r} = \{3; 1; 0\}$
18.  $\bar{a} = \{6; -4; 9\}, \quad \bar{p} = \{1; 0; 2\}, \quad \bar{q} = \{0; 1; 1\}, \quad \bar{r} = \{2; -1; 4\}$
19.  $\bar{a} = \{7; -8; 3\}, \quad \bar{p} = \{3; 1; 0\}, \quad \bar{q} = \{-1; 2; 1\}, \quad \bar{r} = \{-1; 0; 2\}$
20.  $\bar{a} = \{-5; 6; 15\}, \quad \bar{p} = \{-1; 2; 1\}, \quad \bar{q} = \{2; 0; 3\}, \quad \bar{r} = \{1; 1; -1\}$
21.  $\bar{a} = \{7; 11; 6\}, \quad \bar{p} = \{1; 1; 4\}, \quad \bar{q} = \{0; -3; 2\}, \quad \bar{r} = \{2; 1; -1\}$
22.  $\bar{a} = \{-1; -5; 1\}, \quad \bar{p} = \{1; -2; 0\}, \quad \bar{q} = \{-1; 1; 3\}, \quad \bar{r} = \{1; 0; 4\}$
23.  $\bar{a} = \{3; -11; 6\}, \quad \bar{p} = \{1; 0; 5\}, \quad \bar{q} = \{-1; 3; 2\}, \quad \bar{r} = \{0; -1; 1\}$
24.  $\bar{a} = \{4; 3; 10\}, \quad \bar{p} = \{1; 1; 3\}, \quad \bar{q} = \{0; 1; -2\}, \quad \bar{r} = \{1; 0; 3\}$
25.  $\bar{a} = \{1; 5; 7\}, \quad \bar{p} = \{1; 0; 2\}, \quad \bar{q} = \{-1; 0; 1\}, \quad \bar{r} = \{2; 5; -3\}$
26.  $\bar{a} = \{7; -4; 6\}, \quad \bar{p} = \{2; 0; 1\}, \quad \bar{q} = \{1; 1; 0\}, \quad \bar{r} = \{4; 1; 2\}$
27.  $\bar{a} = \{3; 12; 2\}, \quad \bar{p} = \{0; 1; 3\}, \quad \bar{q} = \{1; 2; -1\}, \quad \bar{r} = \{2; 0; -1\}$
28.  $\bar{a} = \{-1; 1; 11\}, \quad \bar{p} = \{1; 2; -1\}, \quad \bar{q} = \{3; 0; 2\}, \quad \bar{r} = \{-1; 1; 1\}$
29.  $\bar{a} = \{-1; 4; 9\}, \quad \bar{p} = \{1; 4; 1\}, \quad \bar{q} = \{-3; 2; 0\}, \quad \bar{r} = \{1; -1; 2\}$
30.  $\bar{a} = \{2; -2; 0\}, \quad \bar{p} = \{0; 1; -2\}, \quad \bar{q} = \{3; -1; 1\}, \quad \bar{r} = \{4; 1; 0\}$