

Здравствуйте, господа студенты!

- У вас на почте в отдельном файле напротив фамилии вы найдёте свой вариант.
  - Пока здесь только начало работы, чтобы вам было, в чём разобраться.
  - Каждое задание 1-14 нужно выполнить на отдельном листе. Наверху должна быть Ваша фамилия и задание. Не забудьте, что кратные интегралы предполагают графики
  - При выполнении заданий 15-22 помните, работа выполняется в Excel, в ОДНОМ файле с названием **Группа\_Фамилия.xlsx**, например, 81ИТ\_Иванов.xlsx, каждое задание на отдельном пронумерованном листе.
- Результаты отправляете мне на почту со своей личной почты, исправления присылаете ответом на моё послание (старайтесь не использовать почту группы, так наша переписка превращается в коллективный чат).
- В 3ей части модуля должны быть ответы на все пункты каждого задания. Пример выполнения ищите в отдельном файле на вашей почте.
  - И самое главное. Задавайте вопросы! Лично или по электронной почте.

1. Представьте двойной интеграл  $\iint_D f(x, y) dx dy$  в виде повторного интеграла

- а) с внешним интегрированием по  $x$ ;  
 б) с внешним интегрированием по  $y$ ,  
 если область  $D$  ограничена указанными линиями.

1.1. $D: x + 2y = 0, x + 6y - 4 = 0, y = 0$	1.19. $D: x = -1, x = -2, y = 0, y = x^2$
1.2. $D: x^2 = 2y, 5x + 2y - 6 = 0$	1.20. $D: y = 0, x^2 = -y, x = \sqrt{1 - y^2}$
1.3. $D: y = \sqrt{8 - x^2}, y = x, y = 0$	1.21. $D: y = 0, y = 1, y = x, x = -\sqrt{2 - y^2}$
1.4. $D: y = 0, x = 0, y = 1, y = \ln x$	1.22. $D: x = 0, y = 1, y = 4, y = -x$
1.5. $D: x^2 + y^2 = 4, x^2 + (y + 2)^2 = 4, y = 0 (x \leq 0)$	1.23. $D: x^2 + y^2 = 9, x^2 + (y - 3)^2 = 9, x = 0 (x \leq 0)$
1.6. $D: y = \sqrt{2 - x^2}, y = x^2$	1.24. $D: x = 0, x = -2, y = x^2 + 4, y = 0$
1.7. $D: y = x^2 - 2, y = x$	1.25. $D: x = 0, y = 0, y = 1, (x - 3)^2 + y^2 = 1$
1.8. $D: y = 1, x = 0, y = 3, y = x$	1.26. $D: x = \sqrt{9 - y^2}, y = x, y = 0$
1.9. $D: y = x^2, y = -x^2 + 2, y = 0 (x \geq 0)$	1.27. $D: x + 2y - 6 = 0, x = y, y = 0$
1.10. $D: y = x, x = 0, y = \sqrt{9 - x^2}$	1.28. $D: 3x + y = 3, y = -x, y = 3$
1.11. $D: y^2 = 2 - x, y = x$	1.29. $D: x = 0, y = 1, y = -1, y = \log_{1/2}(x)$
1.12. $D: x = \sqrt{2 - y^2}, x = y^2, y = 0$	1.30. $D: y = 0, x = 0, y = 1, x = \sqrt{4 - y^2}$
1.13. $D: x = 0, y = \cos x, y = \sin x$	1.31. $D: y = 0, x + 2y - 12 = 0, y = \lg x$
1.14. $D: y = 1, x = 0, y = 3, y = -x$	1.32. $D: x^2 = 2 - y, x + y = 0$
1.15. $D: y = 0, y = x, y = -\sqrt{2 - x^2}$	1.33. $D: y = 3 - x^2, y = -2x$
1.16. $D: y = 0, x = \sqrt{y}, y = \sqrt{6 - x^2}$	1.34. $D: y^2 = 2x, x^2 = 2y, x = 1$
1.17. $D: y = -x, y^2 = x + 3$	1.35. $D: y = \sqrt{4 - x^2}, y = \sqrt{3x}, x = 0$
1.18. $D: x = 1, y = 0, x = 0, y = \sqrt{4 - x^2}$	

2. С помощью двойных интегралов вычислите площадь плоской области  $D$ , ограниченной заданными линиями

2.1. а) $D: y^2=4x, x+y=3, y=0$ б) $D: \begin{cases} x^2+4x+y^2=0, & x^2+6x+y^2=0, \\ y=0, & y=x \end{cases}$	2.19. а) $D: y^2=4x, x=\frac{8}{y^2+4}$ б) $D: \begin{cases} x^2+2x+y^2=0, & x^2+10x+y^2=0, \\ y=0, & y=\sqrt{3}\cdot x \end{cases}$
2.2. а) $D: y=6x^2, x+y=2, x=0$ б) $D: \begin{cases} x^2-6y+y^2=0, & x^2-10y+y^2=0, \\ x=0, & y=-x \end{cases}$	2.20. а) $D: y=4-x^2, y=x^2-2x$ б) $D: \begin{cases} x^2-2y+y^2=0, & x^2-4y+y^2=0, \\ y=-\frac{x}{\sqrt{3}}, & y=-\sqrt{3}\cdot x \end{cases}$
2.3. а) $D: y^2=x+2, x=2$ б) $D: \begin{cases} x^2+2y+y^2=0, & x^2+4y+y^2=0, \\ x=0, & y=\sqrt{3}\cdot x \end{cases}$	2.21. а) $D: x=y^2+1, x+y=3$ б) $D: \begin{cases} x^2-2x+y^2=0, & x^2-6x+y^2=0, \\ y=-\frac{x}{\sqrt{3}}, & y=-\sqrt{3}\cdot x \end{cases}$
2.4. а) $D: x=-2y^2, x=1-3y^2, x=0, y=0$ б) $D: \begin{cases} x^2-4y+y^2=0, & x^2-6y+y^2=0, \\ x=0, & y=-\sqrt{3}\cdot x \end{cases}$	2.22. а) $D: x^2=3y, y^2=3x$ б) $D: \begin{cases} x^2+2x+y^2=0, & x^2+8x+y^2=0, \\ y=\frac{x}{\sqrt{3}}, & y=\sqrt{3}\cdot x \end{cases}$
2.5. а) $D: x=\sqrt{49-y^2}, x=7-\sqrt{49-y^2}$ б) $D: \begin{cases} x^2+2y+y^2=0, & x^2+6y+y^2=0, \\ x=0, & y=-x/\sqrt{3} \end{cases}$	2.23. а) $D: x=\cos y, x=y+1, x=0$ б) $D: \begin{cases} x^2-2x+y^2=0, & x^2-4x+y^2=0, \\ y=0, & y=-x/\sqrt{3} \end{cases}$
2.6. а) $D: y=x^2+1, x+y=3$ б) $D: \begin{cases} x^2-2y+y^2=0, & x^2-10y+y^2=0, \\ y=-\sqrt{3}\cdot x, & y=-x/\sqrt{3} \end{cases}$	2.24. а) $D: x=4-y^2, x-y+2=0$ б) $D: \begin{cases} x^2+2x+y^2=0, & x^2+6x+y^2=0, \\ y=0, & y=x/\sqrt{3} \end{cases}$
2.7. а) $D: y^2=4x, x+y=3, y=0$ б) $D: \begin{cases} x^2+4y+y^2=0, & x^2+10y+y^2=0, \\ y=-\sqrt{3}\cdot x, & y=-x/\sqrt{3} \end{cases}$	2.25. а) $D: x=y^2, x=\sqrt{2-y^2}$ б) $D: \begin{cases} x^2-2x+y^2=0, & x^2-6x+y^2=0, \\ y=0, & y=-x \end{cases}$
2.8. а) $D: y=\cos x, y=x+1, y=0$ б) $D: \begin{cases} x^2-2y+y^2=0, & x^2-4y+y^2=0, \\ x=0, & y=-x \end{cases}$	2.26. а) $D: \frac{x^2}{4}+\frac{y^2}{1}=1, y=\frac{x}{2}, y=0$ б) $D: \begin{cases} x^2+2x+y^2=0, & x^2+4x+y^2=0, \\ y=0, & y=\sqrt{3}\cdot x \end{cases}$
2.9. а) $D: x=\sqrt{4-y^2}, y=\sqrt{3}x, x=0$ б) $D: \begin{cases} x^2+6y+y^2=0, & x^2+8y+y^2=0, \\ x=0, & y=x \end{cases}$	2.27. а) $D: y=-\sqrt{64-x^2}, y=\sqrt{64-x^2}-8$ б) $D: \begin{cases} x^2-4x+y^2=0, & x^2-8x+y^2=0, \\ y=0, & y=-\sqrt{3}\cdot x \end{cases}$
2.10. а) $D: y=x^2+2, x=y, x=2, x=0$ б) $D: \begin{cases} x^2-4y+y^2=0, & x^2-12y+y^2=0, \\ x=0, & y=-x \end{cases}$	2.28. а) $D: y=x^2, y=\frac{3}{4}x^2+1$ б) $D: \begin{cases} x^2+4x+y^2=0, & x^2+8x+y^2=0, \\ y=-x/\sqrt{3}, & y=-\sqrt{3}\cdot x \end{cases}$

2.11. a) $D: y = 4x^2, 9y = x^2, y = 2$ б) $D: \begin{cases} x^2 - 4y + y^2 = 0, & x^2 - 8y + y^2 = 0, \\ x = 0, & y = \sqrt{3} \cdot x \end{cases}$	2.29. a) $D: x = y^2, y^2 = 4 - x$ б) $D: \begin{cases} x^2 - 4x + y^2 = 0, & x^2 - 6x + y^2 = 0, \\ y = -x/\sqrt{3}, & y = -\sqrt{3} \cdot x \end{cases}$
2.12. a) $D: y = \sqrt{49 - x^2}, y = 7 - \sqrt{49 - x^2}$ б) $D: \begin{cases} x^2 - 2y + y^2 = 0, & x^2 - 10y + y^2 = 0, \\ x = 0, & y = -x/\sqrt{3} \end{cases}$	2.30. a) $D: xy = 1, x^2 = y, y = 2$ б) $D: \begin{cases} x^2 + 6x + y^2 = 0, & x^2 + 10x + y^2 = 0, \\ y = x/\sqrt{3}, & y = \sqrt{3} \cdot x \end{cases}$
2.13. a) $D: x = y^2, x = \frac{3}{4}y^2 + 1, y = 0$ б) $D: \begin{cases} x^2 + 4y + y^2 = 0, & x^2 + 8y + y^2 = 0, \\ x = 0, & y = x/\sqrt{3} \end{cases}$	2.31. a) $D: y = \frac{8}{x^2 + 4}, x^2 = 4y$ б) $D: \begin{cases} x^2 - 4x + y^2 = 0, & x^2 - 8x + y^2 = 0, \\ y = x/\sqrt{3}, & y = 0 \end{cases}$
2.14. a) $D: y = \sqrt{2 - x^2}, y = x^2$ б) $D: \begin{cases} x^2 - 6y + y^2 = 0, & x^2 - 8y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x/\sqrt{3} \end{cases}$	2.32. a) $D: x = \sqrt{27 - y^2}, x = 3\sqrt{3} - \sqrt{27 - y^2}$ б) $D: \begin{cases} x^2 - 2x + y^2 = 0, & x^2 - 4x + y^2 = 0, \\ y = -x, & y = 0 \end{cases}$
2.15. a) $D: x = -\sqrt{49 - y^2}, x = \sqrt{49 - y^2} - 7$ б) $D: \begin{cases} x^2 + 8y + y^2 = 0, & x^2 + 10y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x/\sqrt{3} \end{cases}$	2.33. a) $D: y = x^2 + 4x, y = x + 4$ б) $D: \begin{cases} x^2 + 4x + y^2 = 0, & x^2 + 8x + y^2 = 0, \\ y = -x, & y = 0 \end{cases}$
2.16. a) $D: 2y = \sqrt{x}, x + y = 5, x = 0$ б) $D: \begin{cases} x^2 - 10y + y^2 = 0, & x^2 - 12y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x \end{cases}$	2.34. a) $D: y = 2^x, y = 2x - x^2, x = 2, x = 0$ б) $D: \begin{cases} x^2 - 2x + y^2 = 0, & x^2 - 12x + y^2 = 0, \\ y = -x, & y = -x/\sqrt{3} \end{cases}$
2.17. a) $D: x = -\sqrt{27 - y^2}, x = \sqrt{27 - y^2} - 3\sqrt{3}$ б) $D: \begin{cases} x^2 + 2y + y^2 = 0, & x^2 + 12y + y^2 = 0, \\ y = x/\sqrt{3}, & y = x \end{cases}$	2.35. a) $D: y^2 = 4x, x + y = 3, y = 0$ б) $D: \begin{cases} x^2 + 8x + y^2 = 0, & x^2 + 12x + y^2 = 0, \\ y = -x, & y = -\sqrt{3} \cdot x \end{cases}$
2.18. a) $D: y = -2x^2 + 2, y = -6$ б) $D: \begin{cases} x^2 - 4y + y^2 = 0, & x^2 - 12y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x/\sqrt{3} \end{cases}$	2.36. a) $D: y^2 = 4 - x, y = x + 2, y = 2, y = -2$ б) $D: \begin{cases} x^2 - 4x + y^2 = 0, & x^2 - 12x + y^2 = 0, \\ y = -x/\sqrt{3}, & y = -\sqrt{3} \cdot x \end{cases}$

3. Вычислите массу неоднородной пластины  $D$ , ограниченной заданными линиями, если поверхностная плотность в каждой её точке  $\mu = \mu(x, y)$

3.1. $D: y^2 = x, x = 3, \mu(x, y) = x$	3.19. $D: y = x^2, y = 4, \mu(x, y) = 2x + 5y + 10$
3.2. $D: \begin{cases} x^2 + y^2 = 4, & x^2 + y^2 = 100, & x = 0, & y = 0, \\ (x \leq 0, & y \leq 0), & \mu(x, y) = -\frac{7y + x}{x^2 + y^2} \end{cases}$	3.20. $D: \begin{cases} x^2 + y^2 = 1, & x^2 + y^2 = 49, & x = 0, & y = 0, \\ (x \geq 0, & y \geq 0), & \mu(x, y) = \frac{5x + 2y}{x^2 + y^2} \end{cases}$
3.3. $D: y = 0, x = 0, 2x + 3y = 6, \mu(x, y) = y^2 / 2$	3.21. $D: y = 0, x = 0, x + y = 1, \mu(x, y) = x^2$
3.4. $D: \begin{cases} x^2 + y^2 = 1, & x^2 + y^2 = 49, & x = 0, & y = 0, \\ (x \geq 0, & y \geq 0), & \mu(x, y) = \frac{5x + 2y}{x^2 + y^2} \end{cases}$	3.22. $D: \begin{cases} x^2 + y^2 = 9, & x^2 + y^2 = 36, & x = 0, & y = 0, \\ (x \leq 0, & y \geq 0), & \mu(x, y) = \frac{3y - 4x}{x^2 + y^2} \end{cases}$

3.5. $D: y=0, x=0, y=4, x=\sqrt{25-y^2},$ $\mu(x,y)=x$	3.23. $D: y=\pi/2, y=\pi, x=1, x=2,$ $\mu(x,y)=y\sin(xy)$
3.6. $D: x^2+y^2=1, \mu(x,y)=2-x-y$	3.24. $D: y=0, x=0, x+y=2, \mu(x,y)=x^2+y^2$
3.7. $D: x^2+y^2=4y, \mu(x,y)=4-y$	3.25. $D: y=0, x=0, x+y=1, \mu(x,y)=2x^2+y^2$
3.8. $D: x=y, y=-x, y=1, \mu(x,y)=\sqrt{1-y}$	3.26. $D: y\leq 0, x\geq 0, x^2+y^2=4, \mu(x,y)=4-x^2$
3.9. $D: y=2x, x=0, x+y=2, \mu(x,y)=2-x-y$	3.27. $D: y=x^2, y=2, \mu(x,y)=2-y$
3.10. $D: x=1, x=y^2, \mu(x,y)=4-x-y$	3.28. $D: y=0, x=0, x+y=1, \mu(x,y)=x^2+y^2$
3.11. $D: y=0, x^2=1-y, \mu(x,y)=3-x-y$	3.29. $D: y=x^2+1, x+y=3, \mu(x,y)=4x+5y+2$
3.12. $D: y=x^2, x=y^2, \mu(x,y)=3x+2y+6$	3.30. $D: y=x^2+1, x+y=1, \mu(x,y)=2x+5y+8$
3.13. $D: x^2+y^2=4x, \mu(x,y)=4-x$	3.31. $D: y=1, x=0, x=y, \mu(x,y)=x^2+2y^2$
3.14. $D: y=0, y=2x, x+y=6, \mu(x,y)=x^2$	3.32. $D: y=0, x=0, x+y=1, \mu(x,y)=x^2+y^2$
3.15. $D: y^2=1-x, x=0, \mu(x,y)=2-x-y$	3.33. $D: y=x, y=x^2, \mu(x,y)=2x+3y$
3.16. $D: y=\sqrt{x}, x=y, \mu(x,y)=2-x-y$	3.34. $D: x=0, x+2y+2=0, x+y=1, \mu(x,y)=x^2$
3.17. $D: y=0, x=1, x=y,$ $\mu(x,y)=x^2+2y^2+10$	
3.18. $D: y=0, x=0, x+2y=1,$ $\mu(x,y)=2-x^2-y^2$	

4. С помощью тройных интегралов вычислите объём тела  $V$ , ограниченного поверхностями

4.1. $y=x^2, x=y^2, z=3x+2y+6, z=0$	4.19. $y=x^2, y+z=2, y=0, z=0, x=0$
4.2. $z=20-x^2, z=-8x, y=x^2+z^2, y=0$	4.20. $x=\sqrt{z}, x=1/z, z=16, y=4x^2+z^2, y=0$
4.3. $y^2=1-x, x+y+z=2, x=0, z=0$	4.21. $y=3\sqrt{x}, y=3x, x+y+z=10, z=0$
4.4. $y=5-z^2, y=-4z, x=4+y^2+z^2, x=0$	4.22. $x=10+z^2+2y^2, z=y, z=1, y=0, x=0$
4.5. $z=3x^2+2y^2+1, y=x^2-1, y=1, z=0$	4.23. $z=x^2, x+y=6, y=2x, y=0, z=0$
4.6. $y=x+3, x=4, z=\sqrt{25-x^2}, z=0, y=0$	4.24. $z=10+x^2+2y^2, x=y, x=1, y=0, z=0$
4.7. $2x+3y-12=0, 2z=y^2, x=0, y=0, z=0$	4.25. $y=3\sqrt{z}, y=3/z, x=3+y^2+2z^2, x=0, z=5$
4.8. $x=4z^2+3y^2, y=z, y=3z, z=2, x=0$	4.26. $z=4-x^2, x+y^2=4, x=0, z=0$
4.9. $x=1-z^2, x+y+z=3, x=0, y=0$	4.27. $z=2x^2+y^2, x+y=4, x=0, y=0, z=0$
4.10. $y=x^2, x-2z+2=0, x+z-7=0, x=0, y=0$	4.28. $y=1-x^2, x+y+z=3, x=0, y=0, z=0$
4.11. $y=x^3, y=1, x=0, z=2-x^2-y^2, z=0$	4.29. $y=\sqrt{x}, y=x, x+y+z=2, z=0$
4.12. $x+z=2, z^2=4-y, y=0, x=0, z=0$	4.30. $z=x, y=4, x=\sqrt{25-y^2}, x=0, z=0$
4.13. $y=7-z^2, z=x, z=-x, y=0, z=2$	4.31. $z=2x^2+y^2, y=x, y=3x, x=2, z=0, (z\geq 0)$
4.14. $y=2x, x+y=6, x+y+z=20, y=0, z=0$	4.32. $z=2x^2+3y^2, y=x^2, y=x, z=0, (z\geq 0)$
4.15. $y=x^2, y=4, z=2x+5y+10, z=0$	4.33. $z=x^2, x-2y+2=0, x+y-7=0, x=0, z=0$
4.16. $z=2x^2+y^2, z=0, x+y=1, x=0, y=0$	4.34. $z=2-x^2-y^2, x+2y=1, x=0, y=0, z=0$
4.17. $x=y^2, x=1, x+y+z=4, z=0$	4.35. $z=x^2+y^2, x+y=1, x=0, y=0, z=0$
4.18. $x^2=1-y, y=0, x+y+z=3, z=0$	

5. Вычислите объём тела  $V$ ,

а) ограниченного поверхностями, используя переход к цилиндрическим координатам;

б) ограниченного неравенствами, используя переход к сферическим координатам

<p>5.1. а) <math>\begin{cases} 9(x^2 + y^2) = z^2, x^2 + y^2 = 4, x = 0, y = 0, \\ z = 0, (x \geq 0, y \geq 0, z \geq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 49, \\ -x \leq y \leq 0 \\ -\sqrt{\frac{x^2+y^2}{35}} \leq z \leq \sqrt{\frac{x^2+y^2}{3}} \end{cases}</math></p>	<p>5.19. а) <math>\begin{cases} x^2 + y^2 = 16z^2/49, x^2 + y^2 = 4z/7, \\ x = 0, y = 0, (x \geq 0, y \geq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ 0 \leq y \leq -\sqrt{3} \cdot x \\ 3\sqrt{x^2 + y^2} \leq z \end{cases}</math></p>
<p>5.2. а) <math>x^2 + y^2 + z^2 = 4, x^2 + y^2 = 1, (x^2 + y^2 \leq 1)</math></p> <p>б) <math>\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 81, \\ -x \leq y, y \geq 0, \\ -\sqrt{\frac{x^2+y^2}{35}} \leq z \leq \sqrt{\frac{x^2+y^2}{3}} \end{cases}</math></p>	<p>5.20. а) <math>\begin{cases} x^2 + y^2 + z^2 = 16, x^2 + y^2 = z^2, \\ x = 0, y = 0, (x \geq 0, y \geq 0, z \leq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ 0 \leq y \leq \sqrt{3} \cdot x \\ -3\sqrt{x^2 + y^2} \leq z \leq 0 \end{cases}</math></p>
<p>5.3. а) <math>\begin{cases} x^2 + y^2 = 1, x^2 + y^2 = z, x = 0, y = 0, \\ z = 0 (x \geq 0, y \geq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 64, \\ x \leq y \leq \sqrt{3} \cdot x \\ -\sqrt{\frac{x^2+y^2}{3}} \leq z \end{cases}</math></p>	<p>5.21. а) <math>\begin{cases} 36(x^2 + y^2) = z^2, x^2 + y^2 = 9, \\ x = 0, z = 0, (x \leq 0, z \leq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 64, \\ y \geq 0, y \geq \sqrt{3} \cdot x \\ -\sqrt{\frac{x^2+y^2}{63}} \leq z \leq \sqrt{\frac{x^2+y^2}{3}} \end{cases}</math></p>
<p>5.4. а) <math>\begin{cases} x^2 + y^2 = z^2/49, x^2 + y^2 = z/7, \\ x = 0, y = 0, (x \geq 0, y \geq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 36, \\ -x \leq y \leq -\sqrt{3} \cdot x \\ -\sqrt{\frac{x^2+y^2}{3}} \leq z \leq 0 \end{cases}</math></p>	<p>5.22. а) <math>\begin{cases} x^2 + y^2 + z^2 = 16, x^2 + y^2 = 4, \\ (x^2 + y^2 \leq 4) \end{cases}</math></p> <p>б) <math>\begin{cases} 49 \leq x^2 + y^2 + z^2 \leq 100, \\ y \geq 0, y \geq -\sqrt{3} \cdot x \\ -\sqrt{\frac{x^2+y^2}{3}} \leq z \leq \sqrt{\frac{x^2+y^2}{63}} \end{cases}</math></p>
<p>5.5. а) <math>\begin{cases} x^2 + y^2 + z^2 = 4, x^2 + y^2 = 4z^2, \\ x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 49, \\ -x \leq y \leq -\sqrt{3} \cdot x \\ -\sqrt{\frac{x^2+y^2}{3}} \leq z \leq -\sqrt{\frac{x^2+y^2}{24}} \end{cases}</math></p>	<p>5.23. а) <math>\begin{cases} x^2 + y^2 = 4, x^2 + y^2 = 8z, x = 0, y = 0, \\ z = 0 (x \leq 0, y \geq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ 0 \leq y \leq \sqrt{3} \cdot x \\ -3\sqrt{x^2 + y^2} \leq z \leq \sqrt{\frac{x^2+y^2}{99}} \end{cases}</math></p>
<p>5.6. а) <math>\begin{cases} 16(x^2 + y^2) = z^2, x^2 + y^2 = 1, x = 0, y = 0, \\ z = 0, (x \geq 0, y \geq 0, z \geq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 81, \\ x \leq y \leq -\sqrt{3} \cdot x \\ -3\sqrt{x^2 + y^2} \leq z \leq -\sqrt{\frac{x^2+y^2}{15}} \end{cases}</math></p>	<p>5.24. а) <math>\begin{cases} x^2 + y^2 = 4z^2/25, x^2 + y^2 = 2z/5, \\ x = 0, y = 0, (x \geq 0, y \leq 0) \end{cases}</math></p> <p>б) <math>\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ \sqrt{3} \cdot x \leq y \leq 0 \\ -\sqrt{\frac{x^2+y^2}{99}} \leq z \leq 3\sqrt{x^2 + y^2} \end{cases}</math></p>
<p>5.7. а) <math>x^2 + y^2 + z^2 = 16, x^2 + y^2 = 4, (x^2 + y^2 \leq 4)</math></p>	<p>5.25. а) <math>\begin{cases} x^2 + y^2 + z^2 = 4, x^2 + y^2 = 3z^2, \\ x = 0, y = 0, (x \geq 0, y \leq 0, z \geq 0) \end{cases}</math></p>

$6) \begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 64, \\ \sqrt{3} \cdot x \leq y \leq -\sqrt{3} \cdot x \\ 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{15}} \end{cases}$	$6) \begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 81, \\ y \leq \sqrt{3} \cdot x, \quad y \leq \frac{x}{\sqrt{3}} \\ -\sqrt{\frac{x^2 + y^2}{15}} \leq z \leq 0 \end{cases}$
$5.8. a) \begin{cases} x^2 + y^2 = 4, \quad x^2 + y^2 = 4z, \quad x = 0, \quad y = 0, \\ z = 0 \quad (x \geq 0, \quad y \geq 0) \end{cases}$ $6) \begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 36, \\ -\sqrt{3} \cdot x \leq y \leq \sqrt{3} \cdot x \\ z \leq \sqrt{\frac{x^2 + y^2}{15}} \end{cases}$	$5.26. a) \begin{cases} 25(x^2 + y^2) = z^2, \quad x^2 + y^2 = 4, \quad y = 0, \\ x = 0, \quad z = 0, \quad (x \leq 0, \quad z \leq 0, \quad y \geq 0) \end{cases}$ $6) \begin{cases} 49 \leq x^2 + y^2 + z^2 \leq 100, \\ y \geq \sqrt{3} \cdot x, \quad y \geq \frac{x}{\sqrt{3}} \\ -\sqrt{\frac{x^2 + y^2}{15}} \leq z \end{cases}$
$5.9. a) \begin{cases} x^2 + y^2 = z^2 / 81, \quad x^2 + y^2 = z / 9, \\ x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0) \end{cases}$ $6) \begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 49, \\ y \leq -\sqrt{3} \cdot x, \quad y \leq 0, \\ z \leq 3\sqrt{x^2 + y^2} \end{cases}$	$5.27. a) \begin{cases} x^2 + y^2 + z^2 = 9, \quad x^2 + y^2 = 4, \quad y = 0, \\ (x^2 + y^2 \leq 4, \quad y \leq 0) \end{cases}$ $6) \begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ -\sqrt{3} \cdot x \leq y \leq -\frac{x}{\sqrt{3}} \\ 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{48}} \end{cases}$
$5.10. a) \begin{cases} x^2 + y^2 + z^2 = 64, \quad x^2 + y^2 = 8z^2, \\ x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0, \quad z \geq 0) \end{cases}$ $6) \begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 81, \\ y \leq \sqrt{3} \cdot x, \quad y \leq 0, \\ z \leq -3\sqrt{x^2 + y^2} \end{cases}$	$5.28. a) \begin{cases} x^2 + y^2 = 1, \quad x^2 + y^2 = 6z, \quad x = 0, \quad y = 0, \\ z = 0 \quad (x \geq 0, \quad y \leq 0) \end{cases}$ $6) \begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ -\frac{x}{\sqrt{3}} \leq y \leq -\sqrt{3} \cdot x \\ -\sqrt{\frac{x^2 + y^2}{99}} \leq z \leq \sqrt{\frac{x^2 + y^2}{48}} \end{cases}$
$5.11. a) \begin{cases} x^2 + y^2 = z^2, \quad x^2 + y^2 = 81, \quad x = 0, \quad y = 0, \\ z = 0, \quad (x \geq 0, \quad y \geq 0, \quad z \geq 0) \end{cases}$ $6) \begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 64, \\ y \geq \sqrt{3} \cdot x, \quad y \geq -\frac{x}{\sqrt{3}} \\ -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq -\sqrt{\frac{x^2 + y^2}{15}} \end{cases}$	$5.29. a) \begin{cases} x^2 + y^2 = z^2 / 25, \quad x^2 + y^2 = z / 5, \\ x = 0, \quad y = 0, \quad (x \leq 0, \quad y \leq 0) \end{cases}$ $6) \begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 81, \\ y \leq 0, \quad y \leq -\frac{x}{\sqrt{3}} \\ \sqrt{\frac{x^2 + y^2}{63}} \leq z \leq \sqrt{\frac{x^2 + y^2}{3}} \end{cases}$
$5.12. a) \begin{cases} x^2 + y^2 + z^2 = 49, \quad x^2 + y^2 = 4, \quad z = 0, \\ (x^2 + y^2 \leq 4, \quad z \leq 0) \end{cases}$ $6) \begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 36, \\ y \geq -\sqrt{3} \cdot x, \quad y \geq \frac{x}{\sqrt{3}} \\ -\sqrt{\frac{x^2 + y^2}{15}} \leq z \leq \sqrt{\frac{x^2 + y^2}{3}} \end{cases}$	$5.30. a) \begin{cases} x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 8z^2, \\ x = 0, \quad y = 0, \quad (x \leq 0, \quad y \geq 0, \quad z \geq 0) \end{cases}$ $6) \begin{cases} 49 \leq x^2 + y^2 + z^2 \leq 100, \\ y \geq 0, \quad y \geq -\frac{x}{\sqrt{3}} \\ \sqrt{\frac{x^2 + y^2}{80}} \leq z \leq \sqrt{\frac{x^2 + y^2}{35}} \end{cases}$
$5.13. a) \begin{cases} x^2 + y^2 = 25, \quad x^2 + y^2 = 5z, \quad x = 0, \quad y = 0, \\ z = 0 \quad (x \geq 0, \quad y \geq 0) \end{cases}$ $6) \begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 49, \\ y \geq 0, \quad y \geq \sqrt{3} \cdot x, \\ -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq -\sqrt{\frac{x^2 + y^2}{35}} \end{cases}$	$5.31. a) \begin{cases} 4(x^2 + y^2) = z^2, \quad x^2 + y^2 = 25, \\ y = 0, \quad z = 0, \quad (y \geq 0, \quad z \geq 0) \end{cases}$ $6) \begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ \frac{x}{\sqrt{3}} \leq y \leq \sqrt{3} \cdot x, \\ \sqrt{\frac{x^2 + y^2}{99}} \leq z \leq 3\sqrt{x^2 + y^2} \end{cases}$

5.14. a) $\begin{cases} x^2 + y^2 = 4z^2/49, & x^2 + y^2 = 2z/7, \\ x = 0, y = 0, & (x \geq 0, y \geq 0) \end{cases}$ б) $\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 81, \\ y \geq 0, & y \geq -\sqrt{3} \cdot x, \\ -\sqrt{\frac{x^2+y^2}{63}} \leq z \leq -\sqrt{\frac{x^2+y^2}{15}} \end{cases}$	5.32. a) $\begin{cases} x^2 + y^2 + z^2 = 8, & x^2 + y^2 = 4, & y = 0, \\ (x^2 + y^2 \leq 4, & y \geq 0) \end{cases}$ б) $\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ -\frac{x}{\sqrt{3}} \leq y \leq -\sqrt{3} \cdot x, \\ -3\sqrt{x^2 + y^2} \leq z \leq -\sqrt{\frac{x^2+y^2}{99}} \end{cases}$
5.15. a) $\begin{cases} x^2 + y^2 + z^2 = 16, & x^2 + y^2 = 9z^2, \\ x = 0, y = 0, & (x \geq 0, y \geq 0, z \geq 0) \end{cases}$ б) $\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 64, \\ y \geq -\sqrt{3} \cdot x, & y \geq \frac{x}{\sqrt{3}} \\ -\sqrt{\frac{x^2+y^2}{3}} \leq z \leq -\sqrt{\frac{x^2+y^2}{15}} \end{cases}$	5.33. a) $\begin{cases} x^2 + y^2 = 5, & x^2 + y^2 = 2z, & x = 0, y = 0, \\ z = 0 & (x \leq 0, y \geq 0) \end{cases}$ б) $\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 64, \\ 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{3}} \end{cases}$
5.16. a) $\begin{cases} 9(x^2 + y^2) = z^2, & x^2 + y^2 = 4, \\ y = 0, z = 0, & (y \geq 0, z \geq 0) \end{cases}$ б) $\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 36, \\ y \geq \sqrt{3} \cdot x, & y \geq -\frac{x}{\sqrt{3}} \\ -\sqrt{\frac{x^2+y^2}{48}} \leq z \leq -\sqrt{\frac{x^2+y^2}{80}} \end{cases}$	5.34. a) $\begin{cases} x^2 + y^2 = z^2/64, & x^2 + y^2 = z/8, \\ x = 0, y = 0, & (x \geq 0, y \leq 0) \end{cases}$ б) $\begin{cases} 49 \leq x^2 + y^2 + z^2 \leq 100, \\ -x \leq y \leq \sqrt{3} \cdot x \end{cases}$
5.17. a) $\begin{cases} x^2 + y^2 + z^2 = 4, & x^2 + y^2 = 1, & x = 0, \\ (x^2 + y^2 \leq 1, & x \leq 0) \end{cases}$ б) $\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 81, \\ \frac{x}{\sqrt{3}} \leq y \leq x, \\ \sqrt{\frac{x^2+y^2}{35}} \leq z \leq \sqrt{\frac{x^2+y^2}{3}} \end{cases}$	5.35. a) $\begin{cases} x^2 + y^2 + z^2 = 49, & x^2 + y^2 = 6z^2, \\ x = 0, y = 0, & (x \leq 0, y \geq 0, z \leq 0) \end{cases}$ б) $\begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ \sqrt{\frac{x^2 + y^2}{99}} \leq z \end{cases}$
5.18. a) $\begin{cases} x^2 + y^2 = 1, & x^2 + y^2 = 2z, & x = 0, y = 0, \\ z = 0 & (x \geq 0, y \geq 0) \end{cases}$ б) $\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 25, \\ -\frac{x}{\sqrt{3}} \leq y \leq x, \\ -\sqrt{\frac{x^2+y^2}{35}} \leq z \end{cases}$	5.36. a) $\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ 0 \leq y \leq -\frac{x}{\sqrt{3}} \end{cases}$

6. Вычислите массу неоднородного тела  $V$ , ограниченного поверхностями, если плотность в каждой её точке  $\mu = \mu(x, y, z)$

6.1. $V : z = \sqrt{36 - x^2 - y^2}, \quad 9z = x^2 + y^2,$ $\mu(x, y) = \frac{4(x^2 + y^2)}{3}$	6.19. $V : z = \sqrt{25 - x^2 - y^2}, \quad z = \sqrt{(x^2 + y^2)/99},$ $\mu(x, y) = \frac{15(x^2 + y^2)}{7}$
6.2. $V : z = 9\sqrt{x^2 + y^2}, \quad z = 22 - x^2 - y^2,$ $\mu(x, y) = 9 z $	6.20. $V : z = -\sqrt{100 - x^2 - y^2}, \quad z = -6, \quad x^2 + y^2 = 51,$ $(x^2 + y^2 \leq 51), \quad \mu(x, y) = 4 x $

6.3. $V : z = \sqrt{16 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/15},$ $\mu(x,y) = \frac{4(x^2 + y^2)}{3}$	6.21. $V : z = \sqrt{16 - x^2 - y^2}, 6z = x^2 + y^2,$ $\mu(x,y) = 7(x^2 + y^2)$
6.4. $V : z = \sqrt{36 - x^2 - y^2}, z = 2, x^2 + y^2 = 27,$ $(x^2 + y^2 \leq 27), \mu(x,y) = 6z$	6.22. $V : z = \frac{21}{2}\sqrt{x^2 + y^2}, z = \frac{23}{2} - x^2 - y^2,$ $\mu(x,y) = 3 z $
6.5. $V : z = \sqrt{4/9 - x^2 - y^2}, z = x^2 + y^2,$ $\mu(x,y) = \frac{5(x^2 + y^2)}{2}$	6.23. $V : z = \sqrt{9 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/80},$ $\mu(x,y) = 7(x^2 + y^2)$
6.6. $V : z = 12\sqrt{x^2 + y^2}, z = 28 - x^2 - y^2,$ $\mu(x,y) = 3 x $	6.24. $V : z = -\sqrt{81 - x^2 - y^2}, z = -5, x^2 + y^2 = 45,$ $(x^2 + y^2 \leq 45), \mu(x,y) = 3 y $
6.7. $V : z = \sqrt{9 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/8},$ $\mu(x,y) = \frac{5(x^2 + y^2)}{2}$	6.25. $V : z = \sqrt{1 - x^2 - y^2}, 3z/2 = x^2 + y^2,$ $\mu(x,y) = \frac{7(x^2 + y^2)}{5}$
6.8. $V : z = \sqrt{25 - x^2 - y^2}, z = 1, x^2 + y^2 = 21,$ $(x^2 + y^2 \leq 21), \mu(x,y) = 6 z $	6.26. $V : z = -6\sqrt{x^2 + y^2}, z = x^2 + y^2 - 16,$ $\mu(x,y) = 15 z $
6.9. $V : z = \sqrt{64 - x^2 - y^2}, 12z = x^2 + y^2,$ $\mu(x,y) = \frac{3(x^2 + y^2)}{4}$	6.27. $V : z = \sqrt{36 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/63},$ $\mu(x,y) = \frac{7(x^2 + y^2)}{5}$
6.10. $V : z = \frac{9}{2}\sqrt{x^2 + y^2}, z = \frac{11}{2} - x^2 - y^2,$ $\mu(x,y) = 7 y $	6.28. $V : z = \sqrt{64 - x^2 - y^2}, z = 4, x^2 + y^2 = 39,$ $(x^2 + y^2 \leq 39), \mu(x,y) = 2 x $
6.11. $V : z = \sqrt{49 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/48},$ $\mu(x,y) = \frac{3(x^2 + y^2)}{4}$	6.29. $V : z = \sqrt{144 - x^2 - y^2}, 18z = x^2 + y^2,$ $\mu(x,y) = 2(x^2 + y^2)$
6.12. $V : z = \sqrt{49 - x^2 - y^2}, z = 3, x^2 + y^2 = 33,$ $(x^2 + y^2 \leq 33), \mu(x,y) = 5z$	6.30. $V : z = \frac{3}{2}\sqrt{x^2 + y^2}, z = \frac{5}{2} - x^2 - y^2,$ $\mu(x,y) = 13 z $
6.13. $V : z = \sqrt{9 - x^2 - y^2}, 9z/2 = x^2 + y^2,$ $\mu(x,y) = 15(x^2 + y^2)$	6.31. $V : z = \sqrt{9 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/35},$ $\mu(x,y) = 2(x^2 + y^2)$
6.14. $V : z = \frac{15}{2}\sqrt{x^2 + y^2}, z = \frac{17}{2} - x^2 - y^2,$ $\mu(x,y) = 10 z $	6.32. $V : z = \sqrt{121 - x^2 - y^2}, z = 6, x^2 + y^2 = 21,$ $(x^2 + y^2 \leq 21), \mu(x,y) =  x $
6.15. $V : z = \sqrt{4 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/255},$ $\mu(x,y) = 15(x^2 + y^2)$	6.33. $V : z = \sqrt{9 - x^2 - y^2}, 8z = x^2 + y^2,$ $\mu(x,y) = \frac{2(x^2 + y^2)}{9}$
6.16. $V : z = -\sqrt{64 - x^2 - y^2}, z = -1, x^2 + y^2 = 60,$ $(x^2 + y^2 \leq 60), \mu(x,y) = 12 z $	6.34. $V : z = -18\sqrt{x^2 + y^2}, z = x^2 + y^2 - 63,$ $\mu(x,y) =  y $

6.17. $V: z = \sqrt{\frac{16}{9} - x^2 - y^2}, 2z = x^2 + y^2,$ $\mu(x,y) = \frac{15(x^2 + y^2)}{7}$	6.35. $V: z = \sqrt{49 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/195},$ $\mu(x,y) = \frac{2(x^2 + y^2)}{9}$
6.18. $V: z = 3\sqrt{x^2 + y^2}, z = 10 - x^2 - y^2,$ $\mu(x,y) = 6 y $	

7. Вычислите криволинейные интегралы вдоль а) линии  $L$  от точки  $A$  к точке  $B$ ;  
б) контура  $K$  ( $t$  от 0 до  $2\pi$ )

7.1. а) $\int_{MN} (x^2 + 3y^2) dx + (2x^2 - y^2) dy, L: y = \begin{cases} x, & 0 \leq x \leq 1, \\ 2 - x, & 1 < x \leq 2, \end{cases} M(2;0), N(0;0)$ б) $\int_K -x^2 y^3 dx + dy + z dz, K: x = 4 \cos t, y = 4 \sin t, z = 3$
7.2. а) $\int_{MN} 4y dx - 5x dy, L: x^2 + y^2 = 8, (y \leq 0) M(\sqrt{2};0), N(-\sqrt{2};0)$ б) $\int_K x^2 dx + y dy - z dz, K: x = \cos t, y = \frac{\sqrt{2}}{2} \sin t, z = \frac{\sqrt{2}}{2} \cos t$
7.3. а) $\int_{MN} -y dx + 6x^2 dy, L = MN, M(-2;1), N(3;-2)$ б) $\int_K 2y dx - 3x dy + x dz, K: x = 2 \cos t, y = 2 \sin t, z = 2 - 2 \cos t - 2 \sin t$
7.4. а) $\int_{MN} (x - y) dx + (2x + y) dy, L: y = 2\sqrt{x} M(0;0), N(1;2)$ б) $\int_K y dx - x dy + z dz, K: x = \cos t, y = \sin t, z = 2$
7.5. а) $\int_{MN} xy dx + 7y dy, L: x^2 + y^2 = 16, (x \geq 0) M(0;-4), N(0;4)$ б) $\int_K 3y dx - 5x dy + x dz, K: x = 3 \cos t, y = 3 \sin t, z = 3 - 3 \cos t - 3 \sin t$
7.6. а) $\int_{MN} (x^2 + 2y) dx + (y^2 + 2x) dy, L = MN, M(-4;0), N(0;2)$ б) $\int_K 6z dx - x dy + xy dz, K: x = 3 \cos t, y = 3 \sin t, z = 4$
7.7. а) $\int_{MN} (x + 7y) dx + (y - 3x) dy, L: y = \begin{cases} x, & 0 \leq x \leq 1, \\ 2 - x, & 1 < x \leq 2, \end{cases} M(2;0), N(0;0)$ б) $\int_K y dx - x dy + z^2 dz, K: x = \frac{\sqrt{2}}{2} \cos t, y = \frac{\sqrt{2}}{2} \cos t, z = \frac{\sqrt{2}}{2} \sin t$
7.8. а) $\int_{MN} (x + y\sqrt{x^2 + y^2}) dx + (y - x\sqrt{x^2 + y^2}) dy, L: x^2 + y^2 = 81, M(0;-9), N(0;9), x \leq 0$ б) $\int_K (y - z) dx + (z - x) dy + (x - y) dz, K: x = \cos t, y = \sin t, z = 2(1 - \cos t)$

<p>7.9. a) <math>\int_{MN} 3x dx - 2y^2 dy, \quad L = MN, \quad M(4;2), \quad N(2;-4)</math></p> <p>б) <math>\int_K 2z dx - x dy + y dz, \quad K: x = 2\cos t, y = 2\sin t, z = 1</math></p>
<p>7.10. a) <math>\int_{MN} x dx + (x^2 - 4y) dy, \quad L: y = 2x^2 \quad M(0;0), \quad N(1;2)</math></p> <p>б) <math>\int_K x dx + z^2 dy + y dz, \quad K: x = \cos t, y = 2\sin t, z = 2\cos t - 2\sin t - 1</math></p>
<p>7.11. a) <math>\int_{MN} x^2 y dx - xy^2 dy, \quad L: x^2 + y^2 = 1, \quad (x \leq 0, y \geq 0) \quad M(0;1), \quad N(-1;0)</math></p> <p>б) <math>\int_K z dx + y^2 dy - x dz, \quad K: x = \sqrt{2}\cos t, y = 2\cos t, z = \sqrt{2}\sin t</math></p>
<p>7.12. a) <math>\int_{MN} (2 - y) dx + (y - x^2) dy, \quad L = MN, \quad M(-3;-1), \quad N(3;1)</math></p> <p>б) <math>\int_K x dx - 2z^2 dy + y dz, \quad K: x = \frac{\cos t}{2}, y = \frac{\sin t}{3}, z = \cos t - \frac{\sin t}{3} - \frac{1}{4}</math></p>
<p>7.13. a) <math>\int_{MN} (y^2 - 2x) dx + (y^2 + x^2) dy, \quad L: y = \begin{cases} x, &amp; 0 \leq x \leq 2, \\ 4 - x, &amp; 2 &lt; x \leq 4, \end{cases} \quad M(4;0), N(0;0)</math></p> <p>б) <math>\int_K -z dx - x dy + xz dz, \quad K: x = 5\cos t, y = 5\sin t, z = 4</math></p>
<p>7.14. a) <math>\int_{MN} (2xy - y) dx + (x - y^2) dy, \quad L: x^2 + y^2 = 4, \quad (x \leq 0, y \leq 0) \quad M(0;-2), \quad N(-2;0)</math></p> <p>б) <math>\int_K xz dx + x dy + z^2 dz, \quad K: x = \cos t, y = \sin t, z = \sin t</math></p>
<p>7.15. a) <math>\int_{MN} (x + 5y) dx - 7y dy, \quad L = MN, \quad M(0;-1), \quad N(5;0)</math></p> <p>б) <math>\int_K (y - z) dx + (z - x) dy + (x - y) dz, \quad K: x = 4\cos t, y = 4\sin t, z = 1 - \cos t</math></p>
<p>7.16. a) <math>\int_{MN} (2y - x^2) dx + (3x - 4) dy, \quad L: y = 4x^2 \quad M(0;0), \quad N(1;4)</math></p> <p>б) <math>\int_K -x^2 y^3 dx + 2 dy + xz dz, \quad K: x = \sqrt{2}\cos t, y = \sqrt{2}\sin t, z = 1</math></p>
<p>7.17. a) <math>\int_{MN} (x^2 - y) dx + y dy, \quad L: x^2 + y^2 = 9, \quad (x \geq 0) \quad M(0;3), \quad N(0;-3)</math></p> <p>б) <math>\int_K xy dx + x dy + y^2 dz, \quad K: x = \cos t, y = \sin t, z = \sin t</math></p>
<p>7.18. a) <math>\int_{MN} y^2 dx + (4x - y^2) dy, \quad L = MN, \quad M(6;-1), \quad N(0;1)</math></p> <p>б) <math>\int_K (y - z) dx + (z - x) dy + (x - y) dz, \quad K: x = 3\cos t, y = 3\sin t, z = 2(1 - \cos t)</math></p>
<p>7.19. a) <math>\int_{MN} (x - y) dx + (2x + y) dy, \quad L: y = \begin{cases} 2x, &amp; 0 \leq x \leq 1, \\ 3 - x, &amp; 1 &lt; x \leq 3, \end{cases} \quad M(0;0), N(3;0)</math></p>

6) $\int_K 7z dx - x dy + yz dz, \quad K: x = 6\cos t, y = 6\sin t, z = 1/2$
7.20. a) $\int_{MN} (x + y) dx + (x - 2y) dy, \quad L: x^2 + y^2/9 = 1, \quad (x \geq 0, y \leq 0) \quad M(1;0), N(0;-3)$
6) $\int_K x dx - z^2 dy + 4y dz, \quad K: x = 2\cos t, y = 3\sin t, z = 4\cos t - 3\sin t - 3$
7.21. a) $\int_{MN} (5 - x) dx - (3y - 4)dy, \quad L: y = -4\sqrt{x} \quad M(0;0), N(1;-4)$
6) $\int_K -2z dx - x dy + x^2 dz, \quad K: x = \frac{\cos t}{3}, y = \frac{\sin t}{3}, z = 8$
7.22. a) $\int_{MN} (y - 5x) dx + (x + 4y) dy, \quad L = MN, \quad M(-1;3), N(0;5)$
6) $\int_K x dx - 2z^2 dy + y dz, \quad K: x = 3\cos t, y = 4\sin t, z = 6\cos t - 4\sin t + 1$
7.23. a) $\int_{MN} x dx + (x^2 - 4y)dy, \quad L: y = -2x^2 \quad M(0;0), N(1;-2)$
6) $\int_K z dx + x dy + y dz, \quad K: x = 2\cos t, y = 2\sin t, z = 0$
7.24. a) $\int_{MN} (y - x\sqrt{x^2 + y^2}) dx + (x + y\sqrt{x^2 + y^2}) dy, \quad L: x^2 + y^2 = 25, M(-5;0), N(0;5), y \leq 0$
6) $\int_K x dx + 5z^2 dy + 3y dz, \quad K: x = \cos t, y = 3\sin t, z = 2\cos t - 3\sin t - 2$
7.25. a) $\int_{MN} -y dx + 2x^2 dy, \quad L = MN, \quad M(7;-5), N(6;-1)$
6) $\int_K x^2 y^3 dx + 3 dy + y dz, \quad K: x = \cos t, y = \sin t, z = 5$
7.26. a) $\int_{MN} -x dx + (x - y)dy, \quad L: y = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 3 - x, & 1 < x \leq 3, \end{cases} \quad M(0;0), N(3;0)$
6) $\int_K 4y dx - 3x dy + x dz, \quad K: x = 4\cos t, y = 4\sin t, z = 4 - 4\cos t - 4\sin t$
7.27. a) $\int_{MN} (x^2 - y) dx + (x + 2y^2) dy, \quad L: x^2/4 + y^2/9 = 1, \quad (x \leq 0) \quad M(0;-3), N(0;3)$
6) $\int_K (y - z) dx - (x - z) dy + (x - y) dz, \quad K: x = 2\cos t, y = 2\sin t, z = 3 - 3\cos t$
7.28. a) $\int_{MN} (2y + x) dx - xy^2 dy, \quad L = MN, \quad M(4;-2), N(0;0)$
6) $\int_K 2y dx - z dy + x dz, \quad K: x = \cos t, y = \sin t, z = 4 - \cos t - \sin t$
7.29. a) $\int_{MN} (xy - y) dx + x^2/2 dy, \quad L: y = -2\sqrt{x} \quad M(0;0), N(1;-2)$
6) $\int_K x dx + y^2 dy - z dz, \quad K: x = \sqrt{2}\cos t, y = 2\sin t, z = \sqrt{2}\cos t$

7.30. а) $\int_{MN} y dx + 3x dy, L: 9x^2 + y^2 = 1, (y \geq 0) M(1/3;0), N(-1/3;0)$ б) $\int_K x dx - 3z^2 dy + y dz, K: x = \cos t, y = 4 \sin t, z = 3 + 2 \cos t - 4 \sin t$
7.31. а) $\int_{MN} (3y - x^2) dx - 2x^2 dy, L = MN, M(-6;0), N(0;5)$ б) $\int_K -y dx - 2z dy + xy dz, K: x = \sqrt{3} \cos t, y = \sqrt{3} \sin t, z = 3$
7.32. а) $\int_{MN} (2x - 1) dx + (2 + y) dy, L: y = \begin{cases} 3x, & 0 \leq x \leq 1, \\ 4 - x, & 1 < x \leq 4, \end{cases} M(0;0), N(4;0)$ б) $\int_K y/3 dx - 3x dy + x dz, K: x = 2 \cos t, y = 2 \sin t, z = 1 - 2 \cos t - 2 \sin t$
7.33. а) $\int_{MN} (x^2 + y^2) dx + 2(x^2 + y^2) dy, L: x^2/25 + y^2 = 1, (x \geq 0, y \leq 0) M(5;0), N(0;-1)$ б) $\int_K 2x dx + 6z dy + 2y dz, K: x = \sqrt{5} \cos t, y = \sqrt{5} \sin t, z = 6$

8. Представьте число  $z$  в алгебраической, тригонометрической и показательной формах. Возведите число  $z$  в степень, равную  $n+10$ , и решите уравнение  $w^n = z_3$  (корни изобразите на комплексной плоскости, определив примерные углы поворота)

8.1. $z = 3z_1 \cdot z_2 - z_1/z_2, z_1 = 5+2i, z_2 = -3-2i, n=5, z_3 = 3-2i$
8.2. $z = z_1 \cdot z_2 - z_1/z_2, z_1 = 4+2i, z_2 = -3-2i, n=3, z_3 = 3-2i$
8.3. $z = 2z_1 \cdot z_2 - z_1/z_2, z_1 = 5+3i, z_2 = -3-2i, n=4, z_3 = 3-2i$
8.4. $z = 4z_1 \cdot z_2 - z_1/z_2, z_1 = 4-2i, z_2 = -3-2i, n=5, z_3 = -3-4i$
8.5. $z = 3z_1 \cdot z_2 - z_1/z_2, z_1 = -2+5i, z_2 = 4-2i, n=3, z_3 = -3-4i$
8.6. $z = z_1 \cdot z_2 - 2z_1/z_2, z_1 = 5+2i, z_2 = -1+2i, n=4, z_3 = -3-4i$
8.7. $z = z_1 \cdot z_2 - 2z_1/z_2, z_1 = -4+2i, z_2 = -3-2i, n=5, z_3 = -2+3i$
8.8. $z = z_1 \cdot z_2 - 2z_1/z_2, z_1 = -3-4i, z_2 = 5-2i, n=3, z_3 = -2+3i$
8.9. $z = z_1 \cdot z_2 - 2z_1/z_2, z_1 = -2+3i, z_2 = -2-3i, n=4, z_3 = -2+3i$
8.10. $z = z_1 \cdot z_2 - 2z_1/z_2, z_1 = 5-i, z_2 = -4-2i, n=5, z_3 = 4+3i$
8.11. $z = 2z_1 \cdot z_2 - 3z_1/z_2, z_1 = -4+3i, z_2 = -2+3i, n=3, z_3 = 4+3i$
8.12. $z = z_1 \cdot z_2 - 3z_1/z_2, z_1 = 2-5i, z_2 = -1-3i, n=4, z_3 = 4+3i$
8.13. $z = 3z_1 \cdot z_2 - 3z_1/z_2, z_1 = 3+4i, z_2 = -3-2i, n=5, z_3 = 5+2i$
8.14. $z = 2z_1 \cdot z_2 - 3z_1/z_2, z_1 = -3+2i, z_2 = -5-i, n=3, z_3 = 5+2i$
8.15. $z = z_1 \cdot z_2 - 3z_1/z_2, z_1 = 1-3i, z_2 = -1-3i, n=4, z_3 = 5+2i$
8.16. $z = 3z_1 \cdot z_2 + z_1/z_2, z_1 = 5+2i, z_2 = -3-2i, n=5, z_3 = -5+2i$
8.17. $z = z_1 \cdot z_2 + z_1/z_2, z_1 = 4+2i, z_2 = -3-2i, n=3, z_3 = -5+2i$
8.18. $z = 2z_1 \cdot z_2 + z_1/z_2, z_1 = 5+3i, z_2 = -3-2i, n=4, z_3 = -5+2i$

8.19.	$z = 4z_1 \cdot z_2 + z_1/z_2, z_1 = 4-2i, z_2 = -3-2i, n=5, z_3 = -5-2i$
8.20.	$z = 3z_1 \cdot z_2 + z_1/z_2, z_1 = -2+5i, z_2 = 4-2i, n=3, z_3 = -5-2i$
8.21.	$z = z_1 \cdot z_2 + 2z_1/z_2, z_1 = 5+2i, z_2 = -1+2i, n=4, z_3 = -5-2i$
8.22.	$z = z_1 \cdot z_2 + 2z_1/z_2, z_1 = -4+2i, z_2 = -3-2i, n=5, z_3 = 5-2i$
8.23.	$z = z_1 \cdot z_2 + 2z_1/z_2, z_1 = -3-4i, z_2 = 5-2i, n=3, z_3 = 5-2i$
8.24.	$z = z_1 \cdot z_2 + 2z_1/z_2, z_1 = -2+3i, z_2 = -2-3i, n=4, z_3 = 5-2i$
8.25.	$z = z_1 \cdot z_2 + 2z_1/z_2, z_1 = 5-i, z_2 = -4-2i, n=5, z_3 = 2-4i$
8.26.	$z = 2z_1 \cdot z_2 + 3z_1/z_2, z_1 = -4+3i, z_2 = -2+3i, n=3, z_3 = 2-4i$
8.27.	$z = z_1 \cdot z_2 + 3z_1/z_2, z_1 = 2-5i, z_2 = -1-3i, n=4, z_3 = 2-4i$
8.28.	$z = 3z_1 \cdot z_2 + 3z_1/z_2, z_1 = 3+4i, z_2 = -3-2i, n=5, z_3 = -2-4i$
8.29.	$z = 2z_1 \cdot z_2 + 3z_1/z_2, z_1 = -3+2i, z_2 = -5-i, n=3, z_3 = -2-4i$
8.30.	$z = z_1 \cdot z_2 + 3z_1/z_2, z_1 = 1-3i, z_2 = -1-3i, n=4, z_3 = -2+4i$

9. Проверьте, что функция  $u(x,y)$  ( $v(x,y)$ ) является действительной (мнимой) частью аналитической функции  $f(z)$ . Восстановите эту функцию в окрестности точки  $z_0$  по известной части и значению  $f(z_0)$

9.1.	$u(x,y) = x^2 - y^2 - 2y, f(0) = 3i$	9.16.	$v(x,y) = x^2 + x - y^2, f(0) = 5$
9.2.	$v(x,y) = 1 - \frac{y}{x^2 + y^2}, f(1) = 1+i$	9.17.	$u(x,y) = \frac{e^{2x} + 1}{e^x} \cos y, f(0) = 2 - 3i$
9.3.	$u(x,y) = \frac{x}{x^2 + y^2}, f(1) = 1+i$	9.18.	$v(x,y) = \frac{-y}{(x+1)^2 + y^2}, f(0) = 3$
9.4.	$v(x,y) = e^x \cos y, f(0) = 1+i$	9.19.	$u(x,y) = e^{-y} \cos x, f(0) = 1$
9.5.	$u(x,y) = x^3 - 3xy^2 + 1, f(0) = 1+2i$	9.20.	$v(x,y) = 2xy - 2y, f(0) = 1$
9.6.	$v(x,y) = e^{-y} \sin x + y, f(0) = 2$	9.21.	$u(x,y) = 1 - e^x \sin y, f(0) = 1+i$
9.7.	$u(x,y) = y - 2xy, f(0) = -i$	9.22.	$v(x,y) = x^2 - y^2 + 2x + 1, f(0) = i$
9.8.	$v(x,y) = y - \frac{y}{x^2 + y^2}, f(1) = 2$	9.23.	$u(x,y) = \frac{x}{x^2 + y^2} + x, f(1) = 2 - i$
9.9.	$u(x,y) = x^2 - y^2 - 2x + 1, f(0) = 1$	9.24.	$v(x,y) = 3x^2y - y^3 - y, f(0) = 4$
9.10.	$v(x,y) = y + 2xy, f(0) = -2$	9.25.	$u(x,y) = x^3 - 3xy^2 - x, f(0) = -3i$
9.11.	$u(x,y) = e^x(x \cos y - y \sin y), f(0) = 2i$	9.26.	$v(x,y) = 3x^2y - y^3 - y, f(0) = 4$
9.12.	$v(x,y) = x + 2xy, f(0) = 3$	9.27.	$u(x,y) = -2xy - 2y, f(0) = i$
9.13.	$u(x,y) = e^{-y} \cos x + x, f(0) = 1$	9.28.	$v(x,y) = e^x(y \cos y + x \sin y), f(0) = 2$
9.14.	$v(x,y) = x^2 - y^2 - x, f(0) = 4$	9.29.	$u(x,y) = -x^3 + 3xy^2 - 2x, f(0) = -3i$
9.15.	$u(x,y) = \frac{x+1}{(x+1)^2 + y^2}, f(0) = 1$	9.30.	$v(x,y) = \frac{e^{2x} - 1}{e^x} \sin y, f(0) = 2$

10. Вычислите интеграл  $\int_l (kz + m\bar{z} + c + di)dz$  от комплекснозначной функции по дуге  $l$ ,

где  $l$

а) отрезок, соединяющий точки  $z_A$  и  $z_B$  ;

б) замкнутый контур

10.1. $k = -2; m = -3; c = 8; d = -1;$ а) $z_A = 3+2i, z_B = 1-i;$ б) $l^+ :  z =4 ; \operatorname{Re} z = 0$	10.16. $k = -1; m = -3; c = -8; d = 1;$ а) $z_A = -3+2i, z_B = -1-i;$ б) $l^- :  z =3 ; \operatorname{Im} z = 0$
10.2. $k = -2; m = -2; c = 7; d = 1;$ а) $z_A = -4-3i, z_B = -2+i;$ б) $l^- :  z =4 ; \operatorname{Re} z = 0$	10.17. $k = -1; m = -2; c = -7; d = -1;$ а) $z_A = 4-3i, z_B = 2+i;$ б) $l^+ :  z =3 ; \operatorname{Im} z = 0$
10.3. $k = -2; m = -1; c = 7; d = -2;$ а) $z_A = 4+3i, z_B = 2-i;$ б) $l^+ :  z =4 ; \operatorname{Im} z = 0$	10.18. $k = -1; m = -1; c = -7; d = 2;$ а) $z_A = -4+3i, z_B = -2-i;$ б) $l^- :  z =3 ; \operatorname{Re} z = 0$
10.4. $k = -2; m = 1; c = -1; d = 2;$ а) $z_A = 6-i, z_B = -1+4i;$ б) $l^- :  z =4 ; \operatorname{Im} z = 0$	10.19. $k = -1; m = 1; c = 1; d = -2;$ а) $z_A = -6-i, z_B = 1+4i;$ б) $l^+ :  z =3 ; \operatorname{Re} z = 0$
10.5. $k = -2; m = 2; c = -1; d = -3;$ а) $z_A = -6+i, z_B = 1-4i;$ б) $l^+ :  z =2 ; \operatorname{Re} z = 0$	10.20. $k = -1; m = 2; c = 1; d = 3;$ а) $z_A = 6+i, z_B = -1-4i;$ б) $l^- :  z =6 ; \operatorname{Im} z = 0$
10.6. $k = 1; m = 3; c = 2; d = 3;$ а) $z_A = -4-7i, z_B = 6+2i;$ б) $l^- :  z =2 ; \operatorname{Re} z = 0$	10.21. $k = 2; m = 3; c = -2; d = -3;$ а) $z_A = 4-7i, z_B = -6+2i;$ б) $l^+ :  z =6 ; \operatorname{Im} z = 0$
10.7. $k = 1; m = -4; c = 2; d = 4;$ а) $z_A = 4+7i, z_B = -6-2i;$ б) $l^+ :  z =2 ; \operatorname{Im} z = 0$	10.22. $k = 2; m = -4; c = -2; d = -4;$ а) $z_A = -4+7i, z_B = 6-2i;$ б) $l^- :  z =6 ; \operatorname{Re} z = 0$
10.8. $k = 1; m = 4; c = -3; d = -4;$ а) $z_A = 2-6i, z_B = -3+3i;$ б) $l^- :  z =2 ; \operatorname{Im} z = 0$	10.23. $k = 2; m = 4; c = 3; d = 4;$ а) $z_A = -2-6i, z_B = 3+3i;$ б) $l^+ :  z =6 ; \operatorname{Re} z = 0$
10.9. $k = 1; m = -5; c = -3; d = 5;$ а) $z_A = -2+6i, z_B = 3-3i;$ б) $l^+ :  z =8 ; \operatorname{Re} z = 0$	10.24. $k = 2; m = -5; c = 3; d = -5;$ а) $z_A = 2+6i, z_B = -3-3i;$ б) $l^- :  z =9 ; \operatorname{Im} z = 0$
10.10. $k = 1; m = 5; c = 4; d = -5;$ а) $z_A = -5-i, z_B = 1+5i;$ б) $l^- :  z =8 ; \operatorname{Re} z = 0$	10.25. $k = 2; m = 5; c = -4; d = 5;$ а) $z_A = 5-i, z_B = -1+5i;$ б) $l^+ :  z =9 ; \operatorname{Im} z = 0$
10.11. $k = -3; m = -6; c = 4; d = -6;$ а) $z_A = 5+i, z_B = -1-5i;$ б) $l^+ :  z =8 ; \operatorname{Im} z = 0$	10.26. $k = 3; m = -6; c = -4; d = 6;$ а) $z_A = -5+i, z_B = 1-5i;$ б) $l^- :  z =9 ; \operatorname{Re} z = 0$
10.12. $k = -3; m = 6; c = 5; d = 6;$ а) $z_A = 6-3i, z_B = 7+i;$ б) $l^- :  z =8 ; \operatorname{Im} z = 0$	10.27. $k = 3; m = 6; c = -5; d = -6;$ а) $z_A = -6-3i, z_B = 7+i;$ б) $l^+ :  z =9 ; \operatorname{Re} z = 0$
10.13. $k = -3; m = -7; c = 5; d = -8;$	10.28. $k = 3; m = -7; c = -5; d = 8;$

a) $z_A = 6+3i$ , $z_B = 7-i$ ; б) $l^+$ : $ z =10$ ; $\operatorname{Re} z=0$	a) $z_A = -6+3i$ , $z_B = -7-i$ ; б) $l^-$ : $ z =1$ ; $\operatorname{Im} z=0$
10.14. $k=-3$ ; $m=7$ ; $c=-6$ ; $d=8$ ; a) $z_A = -2-7i$ , $z_B = 4+5i$ ; б) $l^-$ : $ z =10$ ; $\operatorname{Re} z=0$	10.29. $k=3$ ; $m=7$ ; $c=6$ ; $d=-8$ ; a) $z_A = 2-4i$ , $z_B = 4+5i$ ; б) $l^+$ : $ z =1$ ; $\operatorname{Im} z=0$
10.15. $k=-3$ ; $m=-8$ ; $c=-6$ ; $d=7$ ; a) $z_A = 2+7i$ , $z_B = -4-5i$ ; б) $l^+$ : $ z =10$ ; $\operatorname{Im} z=0$	10.30. $k=3$ ; $m=-8$ ; $c=6$ ; $d=-7$ ; a) $z_A = -2+7i$ , $z_B = 4-5i$ ; б) $l^-$ : $ z =1$ ; $\operatorname{Re} z=0$

11. а) Найдите все лорановские разложения данной функции  $f(z)$  по степеням  $z$   
б) Найдите разложение функции  $g(z)$  в ряд Лорана в окрестности точки  $z_0$ . Выделите главную и правильную части ряда Лорана

11.1. а) $f(z) = \frac{12z-39}{4z^3+26z^2+30z}$ б) $g(z) = z^2 \cos \frac{\pi}{z-1}$ , $z_0 = 1$	11.16. а) $f(z) = \frac{z^2-4z+5}{4z^3+21z^2-z^4}$ б) $g(z) = z \cos \frac{6z}{z-3i}$ , $z_0 = 3i$
11.2. а) $f(z) = \frac{z-1}{z^3-4z^2-5z}$ б) $g(z) = z^2 e^{\frac{z}{z+1}}$ , $z_0 = -1$	11.17. а) $f(z) = \frac{5-4z}{2z^3-5z^2+2z}$ б) $g(z) = (z+1) \sin \frac{\pi z}{z+1}$ , $z_0 = -1$
11.3. а) $f(z) = \frac{2z+7}{4z^3-2z^2-2z}$ б) $g(z) = z^3 \sin \frac{2z}{z+i}$ , $z_0 = -i$	11.18. а) $f(z) = \frac{2z^2-6z+15}{3z-z^3-2z^2}$ б) $g(z) = z^2 \sin \frac{2z}{z+i}$ , $z_0 = -i$
11.4. а) $f(z) = \frac{-37z+12}{-4z^3+13z^2-3z}$ б) $g(z) = z^4 \cos \frac{2z+i}{2z-i}$ , $z_0 = \frac{i}{2}$	11.19. а) $f(z) = \frac{3z-2}{3z^3-4z^2+z}$ б) $g(z) = (z-2i)^3 e^{\frac{5i}{(z-2i)^2}}$ , $z_0 = 2i$
11.5. а) $f(z) = \frac{-5z^2+11z-5}{3z^2-2z^3-z}$ б) $g(z) = z \cos \frac{z+1}{z-1}$ , $z_0 = 1$	11.20. а) $f(z) = \frac{85z-36}{2z^3+11z^2-6z}$ б) $g(z) = z^2 e^{\frac{z-2}{z+i}}$ , $z_0 = -i$
11.6. а) $f(z) = \frac{4z^2+13z-4}{-3z^3-5z^2+2z}$ б) $g(z) = z \cos \frac{\pi(z-2)}{z}$ , $z_0 = 0$	11.21. а) $f(z) = \frac{13z-2}{3z^3+5z^2-2z}$ б) $g(z) = z^2 \sin \frac{2z+i}{2z-i}$ , $z_0 = \frac{i}{2}$
11.7. а) $f(z) = \frac{-7z^2+7z-3}{7z^3-2z^4-3z^2}$ б) $g(z) = z^2 \sin \frac{8}{(z-4)^2}$ , $z_0 = 4$	11.22. а) $f(z) = \frac{14z-48}{z^3+6z^2-16z}$ б) $g(z) = (z-i)^3 e^{\frac{2i}{(z-i)^2}}$ , $z_0 = i$

11.8. a) $f(z) = \frac{10-6z}{3z^2 - z^3 - 2z}$ б) $g(z) = z^2 \cos \frac{z}{z+2}, z_0 = -2$	11.23. a) $f(z) = \frac{28z+48}{24z - z^3 - 2z^2}$ б) $g(z) = z^2 \cos \frac{z^2 - 2z}{(z-1)^2}, z_0 = 1$
11.9. a) $f(z) = \frac{2-8z}{4z^2 - 3z^3 - z}$ б) $g(z) = z^4 e^{\frac{-3}{(z+2)^2}}, z_0 = -2$	11.24. a) $f(z) = \frac{15z-36}{9z^2 - 2z^3 - 9z}$ б) $g(z) = z \sin \frac{\pi(z-1)}{z+2}, z_0 = -2$
11.10. a) $f(z) = \frac{17z^2 + 10z + 5}{2z^3 + z - 3z^2}$ б) $g(z) = \cos \frac{z^2 - 8z}{(z-4)^2}, z_0 = 4$	11.25. a) $f(z) = \frac{12-z}{z^3 - 7z^2 + 12z}$ б) $g(z) = z^2 e^{\frac{z}{z+1}}, z_0 = -1$
11.11. a) $f(z) = \frac{10z^2 - 16z + 2}{3z^3 - 7z^2 + 2z}$ б) $g(z) = z^2 e^{\frac{\pi}{(z-2)^2}}, z_0 = 2$	11.26. a) $f(z) = \frac{9z^2 + 34z - 15}{z^3 + 2z^2 - 3z}$ б) $g(z) = z^2 \cos \frac{\pi(z+1)}{z}, z_0 = 0$
11.12. a) $f(z) = \frac{5z-4}{5z^2 - 2z - 2z^3}$ б) $g(z) = z^2 \cos \frac{\pi(z+1)}{z}, z_0 = 0$	11.27. a) $f(z) = \frac{-7z+5}{z^3 + 6z^2 + 5z}$ б) $g(z) = z^2 \cos \frac{z+2}{z-4i}, z_0 = 4i$
11.13. a) $f(z) = \frac{2z+1}{z^4 + z^3 - 2z^2}$ б) $g(z) = z \cos \frac{z-3}{z+3}, z_0 = -3$	11.28. a) $f(z) = \frac{13z+7}{6z^2 - z^3 + 7z}$ б) $g(z) = (z-3i)^2 e^{\frac{z}{z-3i}}, z_0 = 3i$
11.14. a) $f(z) = \frac{8z+4}{-9z^3 + 3z^2 + 6z}$ б) $g(z) = z^2 \cos \frac{z}{z+2i}, z_0 = -2i$	11.29. a) $f(z) = \frac{8z^2 - 28z + 50}{2z^3 - 15z^2 + 25z}$ б) $g(z) = z^2 e^{\frac{50}{(z+5i)^2}}, z_0 = -5i$
11.15. a) $f(z) = \frac{z-1}{z^3 + 4z^2 - 5z}$ б) $g(z) = (z+1)^3 \cos \frac{\pi z}{z+1}, z_0 = -1$	11.30. a) $f(z) = \frac{2z-4}{z^4 - 4z^3 - 21z^2}$ б) $g(z) = \sin \frac{i-3z}{2z-7i}, z_0 = 3,5i$

12. Определите тип особой точки  $z = 0$  для функции  $f(z)$

12.1. $f(z) = \frac{e^{2z} - 1 - 2z}{\operatorname{sh} 6z - 6z - 36z^3}$	12.16. $f(z) = \frac{\operatorname{ch} 3z - 1}{\operatorname{sh} 6z - 6z - 36z^3}$
12.2. $f(z) = z^4 \operatorname{ch} \left( \frac{5}{z^3} \right)$	12.17. $f(z) = \frac{e^{z^3} - 1}{\operatorname{ch} 2z - 1 - 2z^2}$
12.3. $f(z) = \frac{\cos 2z - 1 + 2z^2}{e^z - 1}$	12.18. $f(z) = z^2 e^{3/z^5}$

12.4. $f(z) = \frac{\cos 3z - 1}{\operatorname{sh} z - z - z^3/6}$	12.19. $f(z) = \frac{\operatorname{sh} z^3 - z^3}{e^z - 1 - z}$
12.5. $f(z) = \frac{\operatorname{ch} 4z - 1 - 8z^2}{e^z - 1 - z}$	12.20. $f(z) = \frac{\cos z^2 - 1}{\operatorname{sh} z - z - z^3/6}$
12.6. $f(z) = z^2 \sin\left(\frac{2}{z^3}\right)$	12.21. $f(z) = \frac{e^{3z} - 1 - 3z}{\cos 2z - 1 + 2z^2}$
12.7. $f(z) = \frac{\operatorname{sh} 3z - 3z}{\operatorname{ch} z - 1 - z^3/2}$	12.22. $f(z) = z^2 \cos\left(\frac{4}{z^3}\right)$
12.8. $f(z) = \frac{e^{8z} - 1}{\operatorname{sh} 4z - 4z}$	12.23. $f(z) = \frac{\operatorname{sh} z^3 - z^3}{\cos z^2 - 1 + z^4/2}$
12.9. $f(z) = \frac{\operatorname{sh} z^2 - z^2}{\cos 2z - 1 + 2z^2}$	12.24. $f(z) = \frac{\operatorname{ch} 6z - 1 - 18z^2}{\operatorname{sh} 6z - 6z}$
12.10. $f(z) = \frac{\cos z^2 - 1}{z^5}$	12.25. $f(z) = z^5 \operatorname{sh}\left(\frac{2}{z^2}\right)$
12.11. $f(z) = \frac{\operatorname{ch} 8z - 1 - 32z^2}{e^{z^2} - 1}$	12.26. $f(z) = \frac{e^{z^2} - 1 - z^2}{\sin 3z - 3z}$
12.12. $f(z) = \frac{e^{4z} - 1 - 4z}{\cos 6z - 1}$	12.27. $f(z) = z^3 \sin\left(\frac{2}{z^4}\right)$
12.13. $f(z) = z^3 \operatorname{ch}\left(\frac{5}{z}\right)$	12.28. $f(z) = \frac{\cos 2z - 1 + 2z^2}{e^{z^2} - 1}$
12.14. $f(z) = \frac{\cos 9z - 1}{\sin z - z + z^3/6}$	12.29. $f(z) = \frac{\cos 9z - 1}{\operatorname{ch} z - 1 - z^3/2}$
12.15. $f(z) = \frac{\operatorname{sh}(z/2) - z/2}{4z^4}$	12.30. $f(z) = \frac{3\operatorname{sh} z^3 - 3z^3}{e^{4z} - 1 - 4z}$

### 13. Вычислите интеграл с помощью вычетов

13.1. $\oint_{ z-3 =1} \frac{\sin 4z + 3}{z^2(z - \pi)} dz$	13.16. $\oint_{ z-1 =2} \frac{z(z + \pi)}{\sin 2z} dz$
13.2. $\oint_{ z =1} \frac{e^{iz} + 3}{\sin 3zi} dz$	13.17. $\oint_{ z-3/2 =1} \frac{7z(z + \pi)}{\sin 3z(z - \pi)} dz$
13.3. $\oint_{ z-1 =3/2} \frac{\ln(z + 5)}{\sin z} dz$	13.18. $\oint_{ z-\pi =1} \frac{3(z^2 + \pi)^2}{i \sin z} dz$
13.4. $\oint_{ z+1 =1/2} \frac{\operatorname{tg} z + 4}{4z^2 + \pi z} dz$	13.19. $\oint_{ z-\pi =2} \frac{\cos^2 z}{2z \sin z} dz$
13.5. $\oint_{ z+1 =2} \frac{\sin^2 z - 7}{z^2 + 2\pi z} dz$	13.20. $\oint_{ z-1 =2} \frac{z^2 + 3}{(z^2 + 4)\sin \frac{z}{3}} dz$

13.6. $\oint_{ z =\pi/2} \frac{z^2 + z + 5}{\sin z(\pi + z)} dz$	13.21. $\oint_{ z =2} \frac{z^2 + \sin z + 6}{z^2 + \pi z} dz$
13.7. $\oint_{ z-1/2 =1} \frac{e^z + 6}{z(z-1)} dz$	13.22. $\oint_{ z =2} \frac{\sin^2 z}{3z \cos z} dz$
13.8. $\oint_{ z-2 =3} \frac{\cos^2 z + 5}{z^2 - \pi^2} dz$	13.23. $\oint_{ z-3/2 =2} \frac{\sin 2z}{z(z-\pi)\left(z + \frac{\pi}{3}\right)} dz$
13.9. $\oint_{ z-6 =1} \frac{\sin^3 z + 4}{z^2 - 4\pi^2} dz$	13.24. $\oint_{ z-1/2 =1} \frac{5iz(z-i)}{\sin \pi z} dz$
13.10. $\oint_{ z+3/2 =1} \frac{\cos^2 z + 9}{2z^2 + \pi z} dz$	13.25. $\oint_{ z-3/2 =2} \frac{4z}{\sin z} dz$
13.11. $\oint_{ z =1/4} \left( \frac{\ln(e+z)}{z \sin(z+\pi/4)} + 2z \right) dz$	13.26. $\oint_{ z-3/2 =2} \frac{z(\sin z + 3)}{\sin z} dz$
13.12. $\oint_{ z =1} \frac{z^3 - 2i}{\sin 2z(z-\pi)} dz$	13.27. $\oint_{ z =1} \frac{8 + \sin z}{z(z+2i)} dz$
13.13. $\oint_{ z =1/2} \frac{z+2}{z(z^2+1)} dz$	13.28. $\oint_{ z-1-i =5/4} \frac{7}{z^2(z-1)} dz$
13.14. $\oint_{ z-i =3/2} \frac{z-3}{z(z^2+4)} dz$	13.29. $\oint_{ z-1/4 =1/3} \frac{z(z+3)^2}{\sin 2\pi z} dz$
13.15. $\oint_{ z-3 =1/2} \frac{e^z - 1}{\sin z} dz$	13.30. $\oint_{ z-1 =3} \frac{ze^{2z}}{\sin z} dz$

#### 14. Вычислите интеграл, используя теорию функций комплексного переменного

14.1. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+2)(x^2+3)^2}$	14.16. $\int_0^{2\pi} \frac{dx}{10 - 3\sqrt{7} \sin x}$
14.2. $\int_0^{2\pi} \frac{dx}{\sqrt{21} \sin x + 7}$	14.17. $\int_{-\infty}^{+\infty} \frac{(x+2)dx}{(x^2+6)^3}$
14.3. $\int_{-\infty}^{+\infty} \frac{(x^2+1)dx}{(x^2+x+1)^2}$	14.18. $\int_0^{2\pi} \frac{dx}{\sqrt{5} \sin x + 3}$
14.4. $\int_0^{2\pi} \frac{dx}{3 \sin x + 5}$	14.19. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+4)(x^2+1)^2}$
14.5. $\int_{-\infty}^{+\infty} \frac{(x^2+4)dx}{(x^2+9)^2}$	14.20. $\int_0^{2\pi} \frac{dx}{\sqrt{35} \sin x - 6}$

14.6. $\int_0^{2\pi} \frac{dx}{4 - 2\sqrt{3} \sin x}$	14.21. $\int_{-\infty}^{+\infty} \frac{dx}{x^4 + 10x^2 + 9}$
14.7. $\int_{-\infty}^{+\infty} \frac{(x^2 - x + 2)dx}{x^4 + 10x^2 + 9}$	14.22. $\int_0^{2\pi} \frac{dx}{5 - 4\sin x}$
14.8. $\int_0^{2\pi} \frac{dx}{3\sqrt{7} \sin x + 8}$	14.23. $\int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2 + 11)^2}$
14.9. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2 + 9)(x^2 + 4)^2}$	14.24. $\int_0^{2\pi} \frac{dx}{\sqrt{3} \sin x - 2}$
14.10. $\int_0^{2\pi} \frac{dx}{9 - 4\sqrt{5} \sin x}$	14.25. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2 - 10x + 29)^2}$
14.11. $\int_{-\infty}^{+\infty} \frac{(x^2 + 1)dx}{(x^2 + 4x + 13)^2}$	14.26. $\int_0^{2\pi} \frac{dx}{7 + 4\sqrt{3} \sin x}$
14.12. $\int_0^{2\pi} \frac{dx}{4\sin x + 5}$	14.27. $\int_{-\infty}^{+\infty} \frac{(x^2 + 5)dx}{x^4 + 5x^2 + 6}$
14.13. $\int_{-\infty}^{+\infty} \frac{dx}{x^4 + 7x^2 + 12}$	14.28. $\int_0^{2\pi} \frac{dx}{8 - 3\sqrt{7} \sin x}$
14.14. $\int_0^{2\pi} \frac{dx}{9 + 4\sqrt{5} \sin x}$	14.29. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2 + 10)^2 (x^2 + 2)^2}$
14.15. $\int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2 + 3)^2}$	14.30. $\int_0^{2\pi} \frac{dx}{2\sqrt{2} \sin x + 3}$