



**Степенные ряды для элементарных функций и оценки их остатков**

1.  $\exp x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad |R_n(x)| \leq |u_n(x)|, \quad |x| < n + 2;$
2.  $\operatorname{sh} x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}, \quad |R_n(x)| \leq |u_n(x)|/3, \quad |x| \leq n;$
3.  $\operatorname{ch} x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}, \quad |R_n(x)| \leq 2|u_n(x)|/3, \quad |x| \leq n;$
4.  $\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}, \quad |R_n(x)| \leq |u_n(x)|, \quad |x| \leq \frac{\pi}{4};$
5.  $\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}, \quad |R_n(x)| \leq |u_n(x)|, \quad |x| \leq \frac{\pi}{4};$
6.  $\operatorname{arctg} x = \begin{cases} \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}, & |x| < 1; \\ \frac{\pi}{2} \operatorname{sgn} x - \sum_{k=0}^{\infty} (-1)^k \frac{x^{-(2k+1)}}{2k+1}, & |x| \geq 1; \\ |R_n(x)| \leq |u_n(x)|. \end{cases}$

**Варианты заданий для самостоятельного выполнения**

1.  $z(x) = [1 + \operatorname{arctg}(16.7x + 0.1)]^{1/2} / \cos(7x + 0.3),$   
 $x = 0.01(0.005)0.05;$
2.  $z(x) = [1 + \operatorname{arctg}(6.4x + 1.1)]^{1/2} / \sin(2x + 1.05),$   
 $x = 0.01(0.005)0.06;$
3.  $z(x) = \exp(1 + x) \cos \sqrt{1 + x}, \quad x = 0.01(0.005)0.06;$
4.  $z(x) = \sqrt{2x + 0.4} \operatorname{arctg}[\cos(3x + 1)], \quad x = 0.01(0.005)0.06;$
5.  $z(x) = \operatorname{sh}(2x + 0.45)^{1/2} / \operatorname{arctg}(6x + 1) \quad x = 0.01(0.005)0.06;$
6.  $z(x) = \sin(4.5x + 0.6) / (1 + x - 12x^2)^{1/2}, \quad x = 0.1(0.01)0.2;$
7.  $z(x) = [\cos(2.6x + 0.1)]^{1/2} / \exp(1 + x), \quad x = 0.1(0.01)0.2;$
8.  $z(x) = [1 + \operatorname{arctg}(0.8x + 0.2)]^{1/2} \exp(2x + 1), \quad x = 0.1(0.01)0.2;$
9.  $z(x) = \sqrt{\sin(x + 0.74)} \operatorname{sh}(0.8x^2 + 0.1), \quad x = 0.1(0.01)0.2;$
10.  $z(x) = \cos(2.8x + \sqrt{1 + x}) \operatorname{arctg}(1.5x + 0.2), \quad x = 0.1(0.01)0.2;$
11.  $z(x) = \operatorname{ch}(1 + \sqrt{1 + x}) \cos \sqrt{1 + x - x^2}, \quad x = 0.1(0.01)0.2;$
12.  $z(x) = \sqrt{1 + x^2} [\sin(3x + 0.1) + \cos(2x + 0.3)], \quad x = 0.2(0.01)0.3;$
13.  $z(x) = \operatorname{arctg}[\sqrt{0.9x + 1} / (1 - x^2)] + \sin(3x + 0.6),$   
 $x = 0.2(0.01)0.3;$
14.  $z(x) = [\operatorname{arctg} \sqrt{1 + 0.6x}] / \sin(1 + 0.4x), \quad x = 0.2(0.01)0.3;$
15.  $z(x) = \operatorname{sh}[\sqrt{1 + x^2} / (1 - x)] / \sin(x^2 + 0.4), \quad x = 0.2(0.01)0.3;$
16.  $z(x) = \operatorname{ch}[\sqrt{x^2 + 0.3} / (1 + x)] \sin[(1 + x) / (0.6x)],$   
 $x = 0.2(0.01)0.3;$
17.  $z(x) = \sqrt{1 + x} \exp(x + 0.5) \sin(0.3x + 0.7). \quad x = 0.5(0.01)0.6;$
18.  $z(x) = [(1 + x) \exp(x + 0.5) + \sin(x + 0.4)]^{1/2}, \quad x = 0.5(0.01)0.6;$
19.  $z(x) = \operatorname{ch}(2x^2 + \sqrt{x}) / \sin(0.3 + \sqrt{x}), \quad x = 0.5(0.01)0.6;$
20.  $z(x) = \cos(0.5 + \sqrt{x}) / \operatorname{arctg}(1 + 2x\sqrt{x}), \quad x = 0.5(0.01)0.6.$