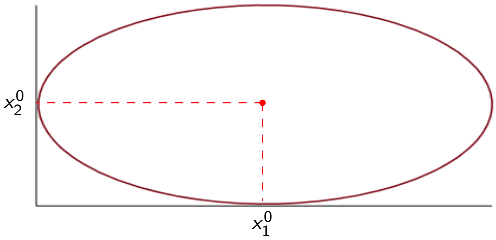
- Linear programming problem: $\min c^\top x \quad \text{s.t.} \quad x \in X = \{x \in \mathbb{R}^n \colon Ax = b,\ x \geqslant 0\}$.

Main property: optimal solution is attained on the boundary of the feasible set X.

- Assumption: a point $x^0 > 0 : \ Ax^0 = b$ is available.

Substitute constraints $x \geqslant 0$ by $x \in E$, where E is the ellipsoid

$$E = \left\{ x \in \mathbb{R}^n : \left(\frac{x_1 - x_1^0}{x_1^0}\right)^2 + \left(\frac{x_2 - x_2^0}{x_2^0}\right)^2 + \cdots + \left(\frac{x_n - x_n^0}{x_n^0}\right)^2 = 1 \right\}.$$



- Linear programming problem: $\min c^\top x \quad \text{s.t.} \quad x \in X = \{x \in \mathbb{R}^n \colon Ax = b,\ x \geqslant 0\}$.

Main property: optimal solution is attained on the boundary of the feasible set X.

- Assumption: a point $x^0 > 0: \ Ax^0 = b$ is available.
- Auxiliary problem $\min c^\top x \quad \text{s.t.} \quad Ax = b, x \in E$ can be solved analytically.

Use previous results with $Q = D^{-1}$,

$$D = \begin{pmatrix} (x_1^0)^2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & (x_2^0)^2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & (x_n^0)^2 \end{pmatrix} = diag((x_1^0)^2, \dots, (x_n^0)^2).$$

In this case $(x - x^0)^\top Q(x - x^0) = \sum\limits_{i=1}^n \left(\frac{x_i - x_i^0}{x_i^0}\right)^2$.

- Dikin interior point algorithm (interior point $x^0 > 0 : Ax^0 = b$ is available).

Step 0. $k = 0, \varepsilon > 0$.

Step 1. Determine $D^k = diag((x_1^k)^2, \ldots, (x_n^k)^2)$.

Step 2. Compute $S^k = AD^k$.

Step 3. Compute $H^k = S^k A^\top$.

Step 4. Compute $h^k = -S^k c$.

Step 5. Solve $H^k \lambda = h^k$. Let λ^k be the solution.

Step 6. Compute $\Delta^k = c + A^\top \lambda^k$.

Step 7. Compute $p^k = D^k \Delta^k$.

Step 8. Compute $r^k = \sqrt{\sum_{i=1}^{n} D_i^k \cdot (p_i^k)^2}$.

Step 9. If $r^k \leqslant \varepsilon$ then stop: x^k is ε-solution.

Step 10. Compute $x^{k+1} = x^k - \frac{1}{r^k} p^k$.

Step 11. Set $k = k + 1$.

Step 12. Goto Step 1.

Example. $\min\{x_1 + 2x_2 + 3x_3\}$ s.t. $x_1 + x_2 + x_3 = 3,\ x_j \geqslant 0,\ j = 1, 2, 3.$

$x^0 = (1, 1, 1)^\top,\ \varepsilon = 0.001, n = 3,\ m = 1,\ A = (1\ 1\ 1),\ b = 3.$

k	x_1^k	x_2^k	x_3^k	$c^\top x^k$	r^k
0	1	1	1	6	1.414
1	1.707	1	0.293	4.586	1
2	2.561	0.293	0.146	3.586	0.411
3	2.869	0.088	0.043	3.175	0.123
4	2.961	0.025	0.013	3.051	0.036
5	2.989	0.008	0.004	3.015	0.011
6	2.997	0.002	0.001	3.004	0.003
7	2.999	0.001	0.000	3.001	0.0009

$x^* = (3, 0, 0),\ c^\top x^* = 3.$