

$$f(t) = \frac{3}{8}t^2 - \frac{1}{3}t + \frac{2}{7}t^3 - \frac{1}{3}t \cdot e^{-2t} + t^4 e^{3t} - \frac{1}{2}t \sin \frac{1}{2}t + \frac{1}{3}e^{3t} \cdot \cos 2t$$

$f(t)$  - оригинал

$L(s)$  - изображение

$$\frac{3}{8}t^2 \xrightarrow{L} \frac{3}{8} \cdot \frac{2}{s^3}$$

$$-\frac{1}{3}t \xrightarrow{L} -\frac{1}{3} \cdot \frac{1}{s^2}$$

$$\frac{2}{7}t^3 \xrightarrow{L} \frac{2}{7} \cdot \frac{6}{s^4}$$

$$-\frac{1}{3}t e^{-2t} \rightarrow -\frac{1}{3} \cdot \frac{1}{(s+2)^2}$$

$$t^4 e^{3t} \rightarrow \frac{4!}{(s-3)^5}$$

$$-\frac{1}{2}t \sin \frac{1}{2}t \xrightarrow{L} -\frac{1}{2} \cdot \frac{2 \cdot \frac{1}{2} \cdot s}{(s^2 + (\frac{1}{2})^2)^2}$$

$$\frac{1}{3}e^{3t} \cos 2t \rightarrow \frac{1}{3} \cdot \frac{s-3}{(s-3)^2 + 2^2}$$

$$L(s) = \frac{3}{4s^3} - \frac{1}{3s^2} + \frac{12}{7s^4} - \frac{1}{3(s+2)^2} + \frac{24}{(s-3)^5} - \frac{s}{2(s^2 + \frac{1}{4})^2} + \frac{s-3}{3((s-3)^2 + 4)}$$



3. Aufgabe 2

$$f(s) = \frac{7s}{49s^2+1} + \frac{1}{4(9s^2+1)} - \frac{4s^2-1}{2(4s^2+1)^2} \quad f(t) = ?$$

$$\frac{7s}{49s^2+1} = \frac{7}{49} \cdot \frac{s}{s^2+\frac{1}{49}} = \frac{1}{7} \cdot \left( \frac{s}{s^2+(\frac{1}{7})^2} \right) \rightarrow \frac{1}{7} \cdot \cos \frac{1}{7}t$$

$$f(t) = \frac{1}{7} \cos \frac{1}{7}t + \frac{1}{12} \sin \frac{1}{3}t - \frac{1}{8}t \cos \frac{1}{2}t$$

$$\frac{1}{4(9s^2+1)} = \frac{1}{36} \cdot \frac{1}{s^2+\frac{1}{9}} = \frac{3}{36} \cdot \frac{\frac{1}{3}}{s^2+(\frac{1}{3})^2} \Rightarrow \frac{1}{12} \cdot \sin \frac{1}{3}t$$

$$\frac{1}{s^2+b^2} \rightarrow \sin bt$$

$$\frac{4s^2-1}{2(4s^2+1)^2} = \frac{4}{2 \cdot 16} \frac{s^2-\frac{1}{4}}{(s^2+\frac{1}{4})^2} = \frac{1}{8} \cdot \frac{s^2-(\frac{1}{2})^2}{(s^2+(\frac{1}{2})^2)^2} \rightarrow \frac{1}{8}t \cdot \cos \frac{1}{2}t$$



Function, $f(t)$	Laplace transform, $F(s)$	Function, $f(t)$	Laplace transform, $F(s)$
1	$\frac{1}{s}$	$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2 + b^2}$
$t$	$\frac{1}{s^2}$	$\sinh bt$	$\frac{b}{s^2 - b^2}$
$t^2$	$\frac{2}{s^3}$	$\cosh bt$	$\frac{s}{s^2 - b^2}$
$t^n$	$\frac{n!}{s^{n+1}}$	$e^{-at} \sinh bt$	$\frac{b}{(s+a)^2 - b^2}$
$e^{at}$	$\frac{1}{s-a}$	$e^{-at} \cosh bt$	$\frac{s+a}{(s+a)^2 - b^2}$
$e^{-at}$	$\frac{1}{s+a}$	$t \sin bt$	$\frac{2bs}{(s^2 + b^2)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$t \cos bt$	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$
$\sin bt$	$\frac{b}{s^2 + b^2}$	$u(t)$ unit step	$\frac{1}{s}$
$\cos bt$	$\frac{s}{s^2 + b^2}$	$u(t-d)$	$\frac{e^{-sd}}{s}$
$e^{-at} \sin bt$	$\frac{b}{(s+a)^2 + b^2}$	$\delta(t)$	1
		$\delta(t-d)$	$e^{-sd}$

$$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$$

3. a) 3

$$\begin{aligned} \mathcal{L}\{f(t)\} &= \frac{gs}{(s+\frac{1}{4})^2 + \frac{1}{4}} = \frac{gs}{16(s+\frac{1}{4})^2 + 4} = \\ &= \frac{g}{16} \cdot \frac{s+\frac{1}{4}-\frac{1}{4}}{(s+\frac{1}{4})^2 + (\frac{1}{2})^2} = \frac{g}{16} \left( \frac{s+\frac{1}{4}}{(s+\frac{1}{4})^2 + (\frac{1}{2})^2} - \frac{\frac{1}{2} \cdot \frac{1}{2}}{(s+\frac{1}{4})^2 + (\frac{1}{2})^2} \right) \\ &= \frac{g}{16} \left( e^{-\frac{1}{4}t} \cos \frac{1}{2}t - \frac{1}{2} e^{-\frac{1}{4}t} \sin \frac{1}{2}t \right) \\ f(t) &= \frac{g}{16} \left( e^{-\frac{1}{4}t} \cos \frac{1}{2}t - \frac{1}{2} e^{-\frac{1}{4}t} \sin \frac{1}{2}t \right) \end{aligned}$$

$$L(s) = \frac{(s-3)^2}{(s^2+1)^2} =$$

$$= \frac{s^2 - 6s + 9}{s^2 (s^2 + (\frac{1}{3})^2)^2} =$$

$$= \frac{s^2 + 9}{s^2 (s^2 + (\frac{1}{3})^2)^2} - \frac{6s}{s^2 (s^2 + (\frac{1}{3})^2)^2}$$

$$= \frac{s^2 - \frac{1}{9}}{s^2 (s^2 + (\frac{1}{3})^2)^2} - \frac{9 \cdot 2s \cdot \frac{1}{3}}{s^2 (s^2 + (\frac{1}{3})^2)^2} +$$

$$+ \frac{\frac{8}{9}}{s^2 (s^2 + (\frac{1}{3})^2)^2}$$

$$= \frac{x^8}{(s+1)^8} \int_0^{\infty} \frac{(s+1)^8}{\Gamma(s)}$$

Преобразование Лапласа

$$L(s) = \int_0^{\infty} e^{-sx} f(x) dx$$

как связаны  $\Gamma(s)$  и  $L(s)$

$$L(s) = \int_0^{\infty} e^{-sx} \cdot F(x) dx$$

$$= -\frac{1}{s} e^{-sx} \cdot F(x) \Big|_0^{\infty} + \frac{1}{s} \int_0^{\infty} e^{-sx} F'(x) dx$$