1) $\xi(t) = \frac{2}{3} t \cdot \eta \cdot \cos\frac{1}{2}t - \frac{1}{4} t \mu \sin\frac{1}{3}t + \frac{t}{3}$

η, μ - независимые с.в.

$\eta \sim Exp(\lambda = \frac{1}{2})$ $\quad \mu \sim Pois(\lambda = 3)$

Найдите: $M\xi(t)$, $D\xi(t)$, $Cov_{\xi}(t_1, t_2)$

Решение

1) $M\xi(t) = M\left(\frac{2}{3} t \eta \cos\frac{t}{2} - \frac{1}{4} t \mu \cdot \sin\frac{t}{3} + \frac{t}{3}\right) =$

$= \frac{2}{3} t \cdot \cos\frac{t}{2} \cdot M\eta - \frac{1}{4} t \cdot \sin\frac{t}{3} M\mu + \frac{t}{3}$

$M\eta = \frac{1}{\lambda} = \frac{1}{1/2} = 2$

$M\mu = \lambda = 3$

$M\xi(t) = \frac{2}{3} t \cdot \cos$

$$M_\xi(t) = \frac{2}{3}t\cdot\cos\frac{t}{2}\cdot 2 - \frac{1}{4}t\cdot\sin\frac{t}{3}\cdot 3 + \frac{t}{3}$$

$$2)\ D_\xi(t) = D\left(\frac{2}{3}t\cdot\eta\cdot\cos\frac{t}{2} - \frac{1}{4}t\cdot\mu\cdot\sin\frac{t}{3} + \frac{t}{3}\right)$$

$$D(a\xi + b\eta + c) = a^2 D\xi + b^2 D\eta + 2ab\,\mathrm{Cov}(\xi,\eta)$$

$$= \left(\frac{2}{3}t\cdot\cos\frac{t}{2}\right)^2\cdot D\eta + \left(-\frac{1}{4}t\cdot\sin\frac{t}{3}\right)^2\cdot D\mu = \frac{4}{9}t^2\cos^2\frac{t}{2}\cdot 4 + \frac{1}{16}t^2\sin^2\frac{t}{3}\cdot 3$$

$$D\eta = 1/\lambda^2 = 1/\left(\frac{1}{2}\right)^2 = 4$$

$$D\mu = \lambda = 3$$

$$\mathrm{Cov}(a_1\xi + b_1\eta + c_1, a_2\xi + b_2\eta + c_2) = a_1a_2 D\xi + b_1b_2 D\eta + (a_1b_2 + b_1a_2)\mathrm{Cov}(\xi,\eta)$$

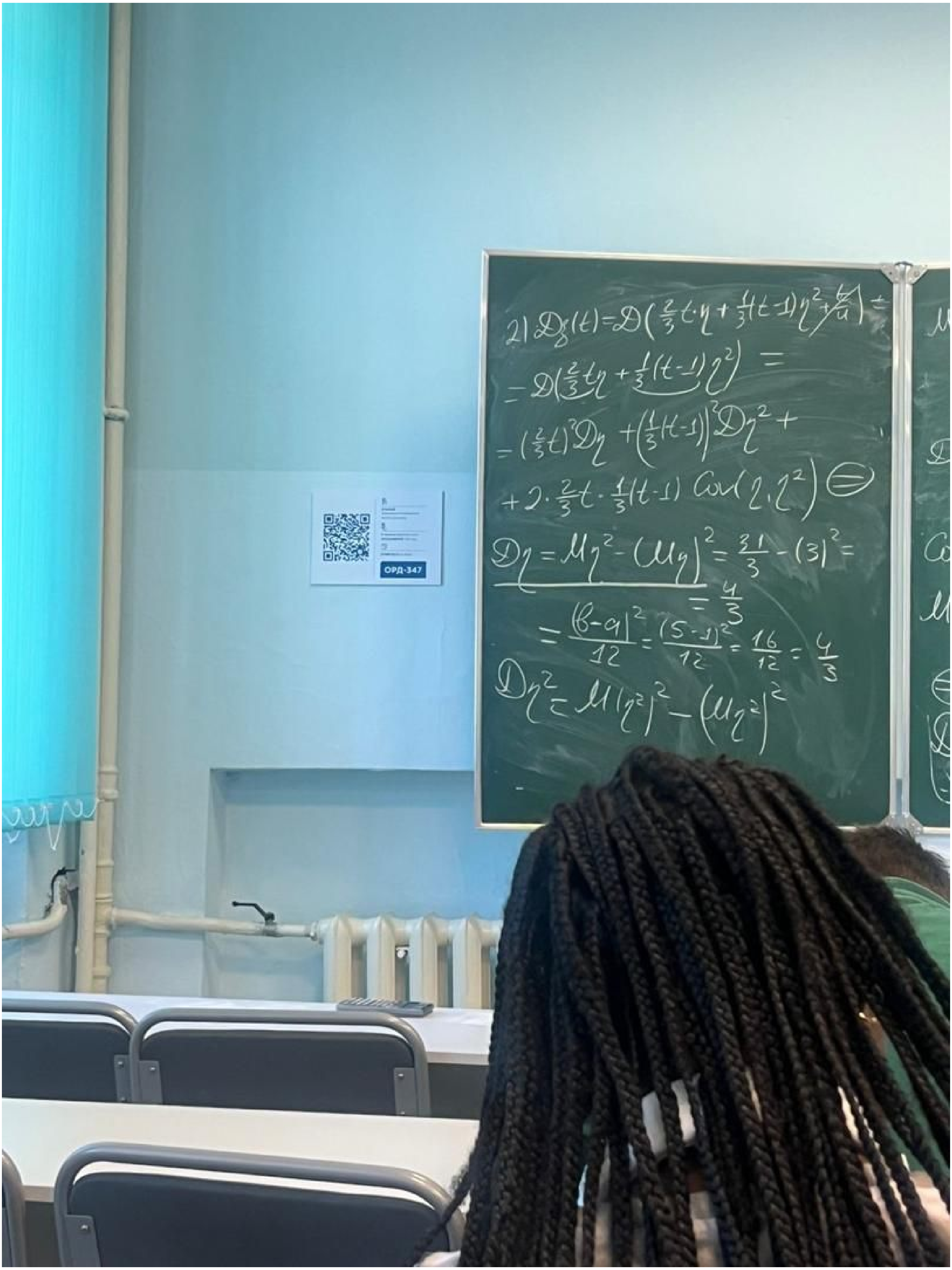
$$\mathrm{Cov}(\xi,\eta) = \rho_{\xi\eta}\cdot\sqrt{D_\xi D_\eta} = \rho\cdot\sigma_\xi\sigma_\eta$$

$$3)\ \mathrm{Cov}_\xi(t_1,t_2) = \mathrm{Cov}(\xi(t_1),\xi(t_2)) =$$

$$= \mathrm{Cov}\left(\frac{2}{3}t_1\eta\cos\frac{t_1}{2} - \frac{1}{4}t_1\mu\sin\frac{t_1}{3} + \frac{t_1}{3},\ \frac{2}{3}t_2\eta\cos\frac{t_2}{2} - \frac{1}{4}t_2\left(\sin\frac{t_2}{3}\right)\mu + \frac{t_2}{3}\right)$$

$$= \left(\frac{2}{3}t_1\cdot\cos\frac{t_1}{2}\right)\cdot\left(\frac{2}{3}t_2\cos\frac{t_2}{2}\right)D\eta + \left(-\frac{1}{4}t_1\sin\frac{t_1}{3}\right)\left(-\frac{1}{4}t_2\sin\frac{t_2}{3}\right)\cdot D\mu$$

$$+\left(\underbrace{\frac{2}{3}t_1\cos\frac{t_1}{2}}_{a_1}\cdot\underbrace{\left(-\frac{1}{4}\right)t_2\sin\frac{t_2}{3}}_{b_2} + \underbrace{\left(-\frac{1}{4}\right)t_1\sin\frac{t_1}{3}}_{b_1}\cdot\underbrace{\frac{2}{3}t_2\cos\frac{t_2}{2}}_{a_2}\right)\mathrm{Cov}(\eta,\mu) = 0$$



$2)\ D\xi(t) = D\left(\frac{2}{3}t\cdot\eta + \frac{1}{3}(t-1)\eta^2 + \cancel{\frac{5}{4}}\right) =$
$= D\left(\frac{2}{3}t\eta + \frac{1}{3}(t-1)\eta^2\right) =$
$= \left(\frac{2}{3}t\right)^2 D\eta + \left(\frac{1}{3}(t-1)\right)^2 D\eta^2 +$
$+ 2\cdot\frac{2}{3}t\cdot\frac{1}{3}(t-1)\, Cov(\eta, \eta^2)\ \ominus$
$D\eta = M\eta^2 - (M\eta)^2 = \frac{31}{3} - (3)^2 = \frac{4}{3}$
$= \frac{(b-a)^2}{12} = \frac{(5-1)^2}{12} = \frac{16}{12} = \frac{4}{3}$
$D\eta^2 = M(\eta^2)^2 - (M\eta^2)^2$
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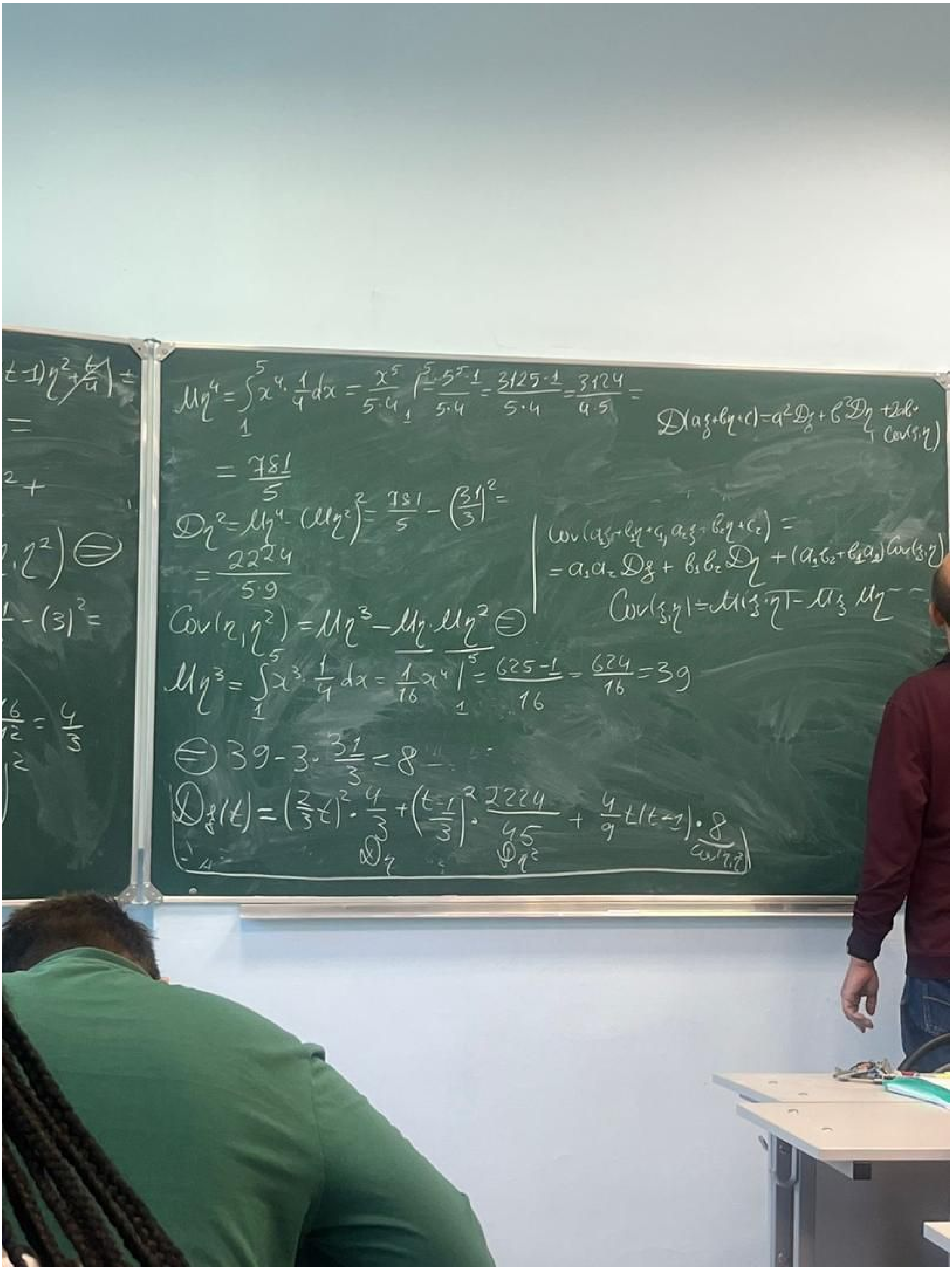

$M\eta^4 = \int_1^5 x^4 \cdot \frac{1}{4}dx = \frac{x^5}{5\cdot4}\Big|_1^5 = \frac{5^5-1}{5\cdot4} = \frac{3125-1}{5\cdot4} = \frac{3124}{4\cdot5} =$
$= \frac{781}{5}$
$D\eta^2 = M\eta^4 - (M\eta^2)^2 = \frac{781}{5} - \left(\frac{31}{3}\right)^2 =$
$= \frac{2224}{5\cdot9}$
$Cov(\eta, \eta^2) = M\eta^3 - M\eta \cdot M\eta^2 =$
$M\eta^3 = \int_1^5 x^3 \cdot \frac{1}{4}dx = \frac{1}{16}x^4\Big|_1^5 = \frac{625-1}{16} = \frac{624}{16} = 39$
$= 39 - 3\cdot\frac{31}{3} = 8$
$D\xi(t) = \left(\frac{2}{3}t\right)^2 \cdot \underbrace{\frac{4}{3}}_{D\eta} + \left(\frac{t-1}{3}\right)^2 \cdot \underbrace{\frac{2224}{45}}_{D\eta^2} + \frac{4}{9}t(t-1)\cdot \underbrace{8}_{Cov(\eta,\eta^2)}$
$D(a\xi+b\eta+c) = a^2D\xi + b^2D\eta + 2ab\,Cov(\xi,\eta)$
$Cov(a_1\xi+b_1\eta+c_1, a_2\xi+b_2\eta+c_2) =$
$= a_1a_2D\xi + b_1b_2D\eta + (a_1b_2+b_1a_2)Cov(\xi,\eta)$
$Cov(\xi,\eta) = M[\xi\cdot\eta] - M\xi\, M\eta$

$D\eta + 2ab \cdot$

$\; + \; Cov(\xi, \eta)$

$b_1 a_2) \, Cov(\xi, \eta)$

$M\eta \sim$

3) $Cov_\xi(t_1,t_2) = Cov(\xi(t_1), \xi(t_2)) = Cov\left(\underbrace{\frac{2}{3}t_1}_{a_1}\eta + \underbrace{\frac{t_1-1}{3}}_{b_1}\eta^2 + \cancel{\frac{t_1}{4}} ; \underbrace{\frac{2}{3}t_2}_{a_2}\eta + \underbrace{\frac{t_2-1}{3}}_{b_2}\eta^2 + \cancel{\frac{t_2}{4}}\right) =$

$$= \frac{2}{3}t_1 \cdot \frac{2}{3}t_2 \underbrace{D\eta}_{4/3} + \frac{(t_1-1)}{3} \cdot \frac{(t_2-1)}{3} \underbrace{D\eta^2}_{2224/45} + \left(\frac{2}{3}t_1 \cdot \frac{t_2-1}{3} + \frac{t_1-1}{3} \cdot \frac{2}{3}t_2\right) \underbrace{Cov(\eta, \eta^2)}_{8} =$$

$$= \left(\frac{4}{9}t_1 t_2 \cdot \frac{4}{3} + \frac{(t_1-1)(t_2-1)}{9} \cdot \frac{2224}{45} + \left(\frac{2t_1(t_2-1)}{9} + \frac{2}{9}t_2(t_1-1)\right) \cdot 8\right) = Cov_\xi(t_1 t_2)$$

Задача 3 $\quad \xi(t) = 2t \cdot |3-\eta| + t, \quad \eta \sim Exp(\lambda = 2)$

Найдите: $M\xi(t)$, $D_\xi(t)$ и $Cov_\xi(t_1,t_2)$

1) $M\xi(t) = M(2t|3-\eta| + t) = 2tM|3-\eta| + t$

$$M|3-\eta| = \int_0^\infty |3-x| \cdot 2e^{-2x}dx = \int_0^3 (3-x)2e^{-2x}dx + \int_3^\infty (x-3)2e^{-2x}dx$$

а) $\int\limits_0^3 (3-x)2e^{-2x}dx = $ $\quad u=3-x \quad du=-dx$
$\quad dv=2e^{-2x}dx \quad v=-e^{-2x}$

$$=(3-x)\cdot(-e^{-2x})\Big|_0^3 - \int\limits_0^3 e^{-2x}dx =$$

$$=3+\frac{1}{2}e^{-2x}\Big|_0^3 = 3+\frac{1}{2}(e^{-6}-1)=$$

$$=\boxed{\frac{5}{2}+\frac{1}{2}e^{-6}}$$

б) $\int\limits_3^\infty (x-3)2e^{-2x}dx=$ $\quad u=x-3 \quad du=dx$
$\quad dv=2e^{-2x}dx \quad v=-e^{-2x}$

$$=\underbrace{-(x-3)e^{-2x}\Big|_3^\infty}_{=0}+\int\limits_3^\infty e^{-2x}dx=$$

$$=-\frac{1}{2}e^{-2x}\Big|_3^\infty = 0+\frac{1}{2}e^{-6}$$

2) $D_\xi(t) = D(2t\cdot|3-\eta| \cancel{+t}) = 4t^2 D|3-\eta| =$

$D|3-\eta| = M(|3-\eta|)^2 - (M|3-\eta|)^2$

$$M(3-\eta)^2 = \int_0^\infty (3-x)^2\cdot 2e^{-2x}dx = \begin{matrix} u=(3-x)^2 & du=-2(3-x)dx \\ dv=2e^{-2x}dx & v=-e^{-2x}\end{matrix}$$

$$= (3-x)^2\cdot(-e^{-2x})\Big|_0^\infty - \int_0^\infty 2(3-x)e^{-2x}dx = 9 - \int_0^\infty (3-x)\cdot 2e^{-2x}dx = \begin{matrix} u=3-x & du=-dx \\ dv=2e^{-2x}dx & v=-e^{-2x}\end{matrix}$$

$$= 9 - \Big(\underbrace{-(3-x)e^{-2x}\Big|_0^\infty}_{3} - \int_0^\infty e^{-2x}dx\Big) = 6 + \int_0^\infty e^{-2x}dx = 6 - \frac{1}{2}e^{-2x}\Big|_0^\infty = 6+\frac{1}{2} = \frac{13}{2}$$

$$D|3-\eta| = \frac{13}{2} - \left(\frac{5}{2} + e^{-6}\right)^2 = \frac{13}{2} - \frac{25}{4} - 5e^{-6} - e^{-12} = \underline{\frac{1}{4} - 5e^{-6} - e^{-12}}$$

$$D_\xi(t) = \left(\frac{1}{4} - 5e^{-6} - e^{-12}\right)\cdot 4t^2$$

3) $Cov_\xi(t_1, t_2) = Cov(\xi(t_1)$

$= 2t_1\cdot 2t_2\, Cov($

Задача 4 $\xi(t) = 2$

$\mu \sim Exp(\lambda=3)$

1) $M_\xi(t) = M($

$M\eta = m = 2$

$Me^{-t(2-\mu)}$

3) $Cov_\xi(t_1, t_2) = Cov(\xi(t_1), \xi(t_2)) = Cov(2t_1 \cdot (3-\eta)(+t_1), 2t_2 \cdot (3-\eta)(+t_2)) =$

$= 2t_1 \cdot 2t_2\, Cov((3-\eta), (3-\eta)) = 4t_1 t_2 \cdot D(3-\eta) = 4t_1 t_2 \left(\frac{1}{4} - 5e^{-6} - e^{-12}\right)$

$u = -dx$

$v = -e^{-2x}$

$6 + \frac{1}{2} = \frac{13}{2}$

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<u>Задача 4</u> $\xi(t) = 2t\eta \cdot e^{-t(2-\mu)}$, где η и μ - независимы, $\eta \sim Norm(m=2, \sigma=3)$

$\mu \sim Exp(\lambda = 3)$ Найдите: $M_\xi(t)$, $D_\xi(t)$, $Cov_\xi(t_1, t_2)$

1) $M_\xi(t) = M\left(2t\eta \cdot e^{-t(2-\mu)}\right) = 2t \cdot M\eta\, M e^{-t(2-\mu)} = \boxed{2t \cdot 2 \cdot \frac{3e^{-2t}}{3-t}}$

$M\eta = m = 2$

$M e^{-t(2-\mu)} = \int_0^\infty e^{-t(2-x)} \cdot 3e^{-3x}\, dx = e^{-2t} \int_0^\infty 3e^{tx-3x}\, dx =$

$= e^{-2t} \int_0^\infty 3e^{(t-3)x}\, dx = \frac{e^{-2t} \cdot 3}{3-t} \int_0^\infty e^{-x(3-t)} \cdot (3-t)\, dx = \frac{3e^{-2t}}{3-t}$